Source estimation for emergency response

Dr Alison Rudd
Department of Meteorology

Stephen Belcher & Alan Robins (University of Surrey)
The problem...
The DYCE consortium

Broadband chemical sensors

Inverse modelling to estimate the source characteristics
Wind tunnel validation experiments
Uncertainty analysis

Communications & networking

Funding
Source Estimation

Inverse problem: estimate source characteristics from concentration measurements

First guess of source characteristics
Strength & location

FORWARD MODEL

Model-predicted concentrations

OPTIMISATION

New estimate of source characteristics

measured concentrations

ITERATION

No – Converged? Yes, best estimate
Forward model - Gaussian plume model

Model used to generate the model-predicted concentrations

Input

- source strength $Q$
- source location $(X_s, Y_s, Z_s)$
- wind speed $u$
- stability
- sensor locations

Assumptions;

- Continuous emission from ground-level source at constant rate
- Steady state flow and constant meteorological conditions
- Neutral conditions
- Dispersion over level, open terrain
Optimisation

An optimal estimate obtained by minimising a cost function starting from a first guess

Cost function, $J(x)$: measures the discrepancy between the measured and model-predicted concentrations

$$J(x) = \frac{1}{2} \sum_{i=1}^{N} \left( \frac{C_i^o - C_i^m}{\sigma_i^2} \right)^2$$

Least squares fit plus error weighting, typically observational and model error

Solve for $J'(x) = 0$

Use Gauss-Newton root finding method

$$x_{k+1} = x_k - \frac{J'}{J''}$$
Estimating the quality of the analysis

\[ p(x) = \exp(-J(x)) \]

\[ \approx \exp\left\{-J(x_0) - \frac{1}{2}x^2 J''(x_0)\right\} \]

Error, or quality estimate:

\[ x_q = \left(\frac{2}{J''(x_0)}\right)^{\frac{1}{2}} \]

For vector of states, \( \mathbf{x} \), use diagonal elements of \( J''(x_0) \)
Wind tunnel measurements
Inverse modelling example

<table>
<thead>
<tr>
<th>Source parameter</th>
<th>True value</th>
<th>First guess</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>0.1</td>
<td>0.7</td>
<td>m³ s⁻¹</td>
</tr>
<tr>
<td>Xs</td>
<td>0</td>
<td>10</td>
<td>m</td>
</tr>
<tr>
<td>Ys</td>
<td>0</td>
<td>10</td>
<td>m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source parameter</th>
<th>Estimate</th>
<th>Uncertainty</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>0.11</td>
<td>0.01</td>
<td>m³ s⁻¹</td>
</tr>
<tr>
<td>Xs</td>
<td>-13.0</td>
<td>13</td>
<td>m</td>
</tr>
<tr>
<td>Ys</td>
<td>4.8</td>
<td>1.0</td>
<td>m</td>
</tr>
</tbody>
</table>

4 sensors:
Mean X distance between source and sensors = 210 m
Mean Y distance between source and sensors = 31 m
Number of sensors

Errors relative to mean distance of sensors to source

Minimum number

Number sensors
4FFID - example time series data

Site C1

Site C2

Site C3

Site C4

Concentration, ppm

Time, min, model scale

Time, min, model scale
Sources of error/uncertainty

• **Measurement error** the accuracy of the concentration measurement from the sensor *may be known*

• **Model error** how good is the model at representing reality? *can only estimate*

• **Sampling error** this is dependent on the averaging time of the data due to the natural variability of the concentrations *likely to dominate*
Shorter time traces

5 mins (WT scale)

5 mins at WT scale is about 10 hours at full scale!
Sampling error

Quantify sampling error:
Short time average to estimate the true mean in a turbulent flow

Standard deviation of the shorter time mean estimate of the true mean concentration

\[
\sigma_{\bar{C}^t} = \left( \frac{1}{n} \sum_{i=1}^{n} \left( \bar{C}_i^t - \bar{C}^T \right)^2 \right)^{1/2}
\]

- \( \bar{C}_i^t \) is the mean concentration averaged over time \( t \)
- \( \bar{C}^T \) is the true mean concentration
- \( t \) is the shorter averaging time
- \( T \) is the total time length
- \( n \) is the number of shorter averaging time samples
Sampling error

$$\frac{\sigma_{C'}}{C}$$ = the uncertainty in the short time mean estimate compared to the true mean concentration

60% uncertainty on 1 min average

70% uncertainty on 10 sec average

20% uncertainty on 15 min average

$$\propto \left(\frac{T_L}{t}\right)^{\frac{1}{2}}$$

Wind tunnel
$$U_{ref} = 2.5 \text{ m/s}$$
$$H = 1m$$

Equivalent full scale
$$U_{ref} = 10 \text{ m/s}$$
$$H = 500m$$
Ensemble

Ensemble of shorter averaged wind tunnel data and effect on the estimate of the source characteristics

Error bars represent the uncertainty of the best estimate

Each line is a different ensemble member

Model error also large at short times
Conclusions

• Developed and tested inverse algorithm
  – Need at least 3 sensors; Better with 5-7 sensors
  – performs best for configurations with sensors across the plume
  – ideally at least 3 sensors across the plume

• Sampling error is likely to be dominant
  – Pressure for quick answers; Perhaps do not need to be accurate?
  – Do need to be consistent with longer term averages

• Future
  – Need to quantify source errors when sampling error large
  – Develop method with network model
  – Use inverse framework for model parameter estimation
  – Intelligent algorithms for iterating sensors placement
  – Unsteady sources - sampling issues even more critical
\[ C_{i,j} = a \, C_{i-1,j} + b \, C_{i,j-1} + c \, C_{i-1,j-1} + Q_{i,j}. \]

Figure 7: Comparison of concentration profile from a DNS with the network model solution. (a) \( \log_{10} C_{i,j}^{DNS} \) for the DNS, (b) \( \log_{10} C_{i,j}^{NM} \) for the network model, with \( a = b = 0.25, r = s = 1 \) and \( c = 0.15 \), normalised difference \( (C_{i,j}^{NM} - C_{i,j}^{DNS})/C_{i,j}^{DNS} \). Concentrations are normalised by that in the first cell, where the source is located. The wind forcing is at 45°, along the leading diagonal.
Comparison with DNS and WT data

Branford et al, 2011

Blue line: DNS
Black circles: WT data
Red triangles: NM

45 deg flow

\[ a = b = 0.25 \]
\[ c = 0.2 \]

\[ \log(C/C_0) \text{ vs. } i \text{ for different } j \]

CO is concentration in source cell; \( i \) and \( j \) are node indices (\( i = j = 0 \) denotes the source cell)