Concentration Fluctuations arising from inherent variability in the field of turbulence.

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Some of the difference between predicted and observed concentrations of a pollutant emitted from a single source must arise from the natural variations in the field of turbulence in time and space.

To explore this, this note considers the simplest of situations: a point source emitting a passive neutral pollutant at a constant rate into an airstream of constant velocity with homogeneous, stationary turbulence. It is assumed that the growth of both the instantaneous plume and the time-averaged plume is linear with distance between the source and the receptor (thus limiting the distance to 1 or perhaps 2 kilometres). Whether the receptor is in or out of the instantaneous plume then depends on the values of the vertical and lateral turbulent velocities $w$ and $v$ experienced by that bit of the plume as it left the source.

A time sequence of $v$'s and $w$'s are generated using the stochastic random-walk equation. These sequences are then sampled over specified durations to see what fraction of time the receptor was in the instantaneous plume and hence what its average concentration would have been. Looking at a great number of such samples yields the probability distribution for such concentrations.

The probability distribution is shown to depend on the sampling duration, the relative magnitudes of the instantaneous and time-averaged plume growth rates, and the position of the receptor (how far it is off the downwind axis through the source).

1 INTRODUCTION

In some situations, comparisons show significant discrepancies between observed and model-predicted concentrations in the plume downwind of a source. This lack of accurate predictability can arise in several ways: insufficient knowledge of the emission rate and temperature, source size, complex building and terrain effects, inadequate models, the use of meteorological parameters (such as wind speed and direction, stability and turbulence) not strictly equating to those affecting the plume, and inherent variability in time within the turbulence itself. This note explores the importance of the last of these causes.

The general approach has been outlined in the Abstract; sufficient here to add that although the investigation has been carried out for a rather ideal scenario,
the general conclusions are likely to apply in nature if not in exact magnitude to more realistic situations in the atmospheric boundary layer.

To reiterate the assumptions in this study, they are now listed:

a. Uniform, homogeneous, stationary turbulence
b. Statistically constant mean wind speed and direction
c. Neutral stability
d. A true point source
e. No barriers to dispersion (e.g. no ground or inversion effects)
f. A pollutant consisting of non-buoyant, passive, neutral particles.
g. Both the instantaneous and time-averaged plumes grow linearly with distance between the source and receptor, in virtual accord with theory and observation out to one or two kilometres.
h. The centroid of each section of the instantaneous plume travels in a virtually straight line dependent on the turbulent velocities $v$ and $w$ experienced at the moment of release at the source.
i. Although in reality there is significant concentration structure within the instantaneous plume, over the time-interval that the plume is “covering” the receptor this structure is smoothed out and the receptor simply experiences the average concentration in the instantaneous plume at that range.

2 THE MODEL

Under the above assumptions the concentration experienced by the receptor averaged over a specified sampling duration $T$ depends on the sequence of turbulent velocities $v$ and $w$ at the source a time $x/u$ earlier ($x$ being the downwind distance between source and receptor, and $u$ being the wind-speed).

It is assumed the turbulent velocities (e.g. $v$) at the source follow a stochastic “random-walk” equation:

$$v(t + \Delta t) = R(\delta t)v(t) + (1 - R^2(\delta t))^{1/2}\sigma, \varepsilon$$

where $R$ is the Eulerian time correlation function. (see Pasquill and Smith, p.133ff), and $\varepsilon$ is a random number with zero mean and unit standard deviation. The equation is solved on a computer using Basic in which random values of $\varepsilon$ are internally generated.

$R(\delta t)$ is made equal to $(1 - \delta t/\tau)$ in accord with an exponential correlation function, provided $\delta t/\tau$ is small enough. Here we put $\delta t/\tau = 0.01$. $\tau$ is the Eulerian timescale.

A time series of $v(t)$ and $w(t)$ ending over a total duration of $2000\tau$ is then sampled (with overlapping samples) over prescribed sampling durations extending from $0.25\tau$ to $64\tau$ to give statistics of concentration.
If in each time-step $\sqrt{v^2 + w^2} > 0.05u$ then the instantaneous plume will not cover the receptor when it reaches $x$ and the concentration is put zero, otherwise it is 1 (i.e. the mean concentration in the instantaneous plume at a downwind distance $x$). The 1’s and 0’s are added to give the overall score or “concentration” for that sample. If this score, divided by the number of time-steps in the sample duration, is 0 it is recorded in “box 0”, if it lies between 0 and 0.1 (but not precisely 0) it is recorded in “box 1”, if it lies between 0.1 and 0.2 it is recorded in “box 2” and so on up to a value between 0.9 and 1 which is recorded in “box 10”. Samples lying in box 0 mean the receptor never was within the instantaneous plume, whereas those in box 10 mean the receptor was within the instantaneous plume for at least 90% of the sample-time.

3 VALUES OF THE PARAMETERS

In the simple homogeneous conditions chosen for the analysis there is no obvious way for prescribing the values of the parameters, so values more appropriate to the lower boundary layer will be used.

The values of $\sigma_v$ and $\sigma_w$, the root-mean-square values of the lateral and vertical components of the turbulent velocities, will be given several values, but the normal values will be both equal to 0.1u.

If $i$ is the intensity of turbulence, equal to the ratio $\sigma_w/u$, then the rate of growth of the instantaneous plume is given by:

$$\frac{d\sigma}{dx} = \frac{2}{3} \beta v^2 \approx \frac{2}{3} \cdot 4.01$$

Consequently $\sigma \approx 0.03x$. The width $W$ is then given by $W \approx 2\sqrt{3} \cdot 0.03x$ if the concentration is statistically uniform across the instantaneous plume.

The band of $w$ corresponding to the total width of the instantaneous plume is therefore $\delta w = W \frac{u}{x} = 0.06\sqrt{3}u \approx 0.1u$.

Hardest to prescribe is the Eulerian time-scale $\tau$. Most of the results do not require $\tau$ to be known, although to apply the results as in the final example value is needed. Here $\tau$ is put equal to 2 minutes.
4 RESULTS

Figure 1 shows the percentage frequency distribution of the “concentration” (i.e. the average number of 1’s in the samples divided by the maximum possible number of 1’s) for samples of normalised duration $\frac{T_s}{\tau}$ ranging from 0.25 to 64.

It can be seen that as the normalised duration gets smaller and smaller the more individual samples are almost all either full of 1’s or 0’s, i.e. the receptor is within the instantaneous plume throughout the short sample or totally outside it. In contrast for very large sample durations, each individual sample has experienced sufficient values of $v$ and $w$ that the fractional number of 1’s, or the fraction of the maximum possible concentration, is virtually the same as in every other sample. The peak percentage frequency then becomes 100% at the mean concentration $\overline{C}$. Intermediate values of $\frac{T_s}{\tau}$ show frequency distributions lying between these two extremes.

In these terms it becomes rather simple to explain the origins of the fluctuations in concentration arising from turbulence.

Figure 1 was for a receptor lying on the downwind axis through the source. Figure 2 illustrates the consequences of having the receptor lying off the axis. A refers to the receptor lying on the axis as in Figure 1, B refers to a receptor located at a point $\sigma_v x/u$ off the axis, and C at a point $2\sigma_v x/u$ off. The graph plots the percentage frequency of individual samples with concentrations lying outside a factor of 2 of the mean against the normalised sample duration for the three receptor locations A, B and C. Thus for $\frac{T_s}{\tau}$ equal to 2, A has 31% of concentrations outside a factor of 2 of the mean, B has 51% and C has 79%. Receptor location is therefore a very important factor in the uncertainty in estimating $\overline{C}$ from measured values.

Figure 3 is similar to Figure 1 except that it is for a single value of $\frac{T_s}{\tau} = 4$, and is for the three receptor locations A, B and C. As the receptor moves further away from the downwind axis the more $\overline{C}$ decreases and the more samples have all 0’s in them.

Figure 4 shows the effect of increasing the values of $\sigma_v$ and $\sigma_w$ and hence the size of the time-averaged plume, leaving the size of the instantaneous plume unchanged. In these circumstances it is reasonable to expect that as the time-averaged plume increases the less often will the receptor be covered by the instantaneous plume. This is clearly confirmed by the curves in this Figure.
**5 AN EXAMPLE**

Suppose we are asked “What is the probability that the concentration at a receptor 100m downwind of a source exceeds 1mg/m\(^3\) given that the source strength is 1g/s, the concentration is to be measured over 30 minutes and the wind speed is 5m/s. An estimate of the Eulerian time-scale is 2 minutes”.

We can approach answering the question by using the above model:

The instantaneous plume width \(W\) is approximately equal to 0.1\(x\) which means 10m at \(x=100m\).

The mean concentration in the instantaneous plume is therefore

\[
C = \frac{4Q}{(u\pi)^{1/2}} = 0.002546 \text{ g/m}^3
\]

since \(Q\), the source strength is 1 g/s and \(u=5\text{m/s}\).

We further assume according to the model that \(\sigma_v = \sigma_u \approx 0.5\text{ m/s}\).

According to Figure 1 the probability that the 30-minute (\(\approx 16\) times the time-scale) average “concentration” exceeds \((1\text{mg/m}^3)/(2.546 \text{ mg/m}^3) \approx 0.4\) is 15%. This is the required answer.

If the sampling duration were reduced to 8 minutes the probability would be increased to 25.5%.

Note that the average concentration at the receptor is 0.816 mg/m\(^3\).

**6 CONCLUSIONS**

This very simple approach has given insight into the way that a limited number of observations of concentration downwind of a source are affected by the inherent variability in turbulence. It shows how sample duration, receptor location and the relative magnitudes of the time-averaged plume and the instantaneous plume all play their part in this uncertainty.

**7 REFERENCES**

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The probability distribution of concentrations as a function of sample duration for a receptor on the source axis.

Fraction of maximum possible concentration
1 means the receptor is always in the plume during the sample
0 means the receptor is never in the plume during the sample.

Figure 1
The percentage number of concentrations within samples outside a factor of 2 of the mean for different sample-durations and different receptor positions. ($\sigma_v = \sigma_z = 0.1 \text{ u}$)

A: receptor on the downwind axis
B: receptor at a point $\sigma_x x/u$ off the axis.
C: receptor at a point $2\sigma_x x/u$ off the axis.

$T_x$ is the sample duration, $\tau$ is the Eulerian time-scale.

Note the "exponential" scale.

Figure 2
Concentration Fluctuations arising from inherent variability in the field of turbulence

The effect of receptor position on the concentration
Probability distribution for a sample duration
\( T_s = 4 \tau \). (A, B, C have the same meaning as before)

fraction of maximum possible concentration
1 means the receptor is always in the plume during the sample
0 means the receptor is never in the plume during the sample.

Figure 3
The effect of increasing $\sigma_v$ and $\sigma_w$.

A has $\sigma_v = \sigma_w = 0.1$ u

B has $\sigma_v = 0.15$ u, $\sigma_w = 0.1$ u

C has $\sigma_v = \sigma_w = 0.15$ u

leaving the instantaneous plume size unchanged.

**Sampling duration**

$= 64.7$

fraction of maximum possible concentration

1 means the receptor is always in the plume during the sample

0 means the receptor is never in the plume during the sample.

Figure 4