

# Teaching mental mathematics from level 5: Number

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These resources can be used to help improve pupils' mathematical thinking skills in number. The teaching approaches cover 'hard to teach' aspects of number, including rounding, fractions, decimals, percentages, and ratio and proportion.

Use these teaching activities to design revision or intervention tasks for pupils who are struggling to make connections between different aspects of number. These activities can also help you develop pupils' mental maths images and mathematical talk through in paired or group work.

## Topics covered

This resource contains teaching approaches that can be used to develop mental maths abilities beyond level 5. These are selected from the Number (numbers and the number system) section of the [Secondary mathematics learning objectives](#).

This includes some of the aspects of number that have been reported as difficult to teach and hard to learn, such as:

- place value and ordering
- rounding
- equivalence between fractions, decimals and percentages
- proportional reasoning to solve problems.

## Benefits of using this resource

The suggested activities in this resource will help you to plan sequences of lessons that provide opportunities for pupils to:

- develop an understanding of place value, including standard form
- decide when it is appropriate to round numbers and how this will affect the result
- work effectively with the equivalences between fractions, decimals and percentages
- understand the role of multiplication in proportional reasoning.

The tasks may easily be adapted to adjust the level of challenge and keep pupils at the edge of their thinking.

## Useful teaching strategies

You can use teaching strategies such as modelling, classifying and matching while teaching aspects of number.

Pupils still need to use and refine their mental calculation strategies, even when they are using written or calculator methods for working out more complex calculations.

A secure understanding of place value underpins calculation and enables pupils to work with both large and small numbers.

In calculation up to level 5, many pupils use a mental image of the number line to

support their strategies. Throughout Key Stage 3, as pupils extend their understanding of the number system, it can be helpful to focus more attention on the structure of the base-10 number system as shown on a place-value chart. This provides a good foundation for work beyond level 5, for example, when using standard form.

In this resource, three teaching strategies are used repeatedly. These are modelling, classifying and matching.

## Modelling

Modelling provides an opportunity for you to make explicit the skills, processes and links that would otherwise be hidden from pupils or unclear to them. Share your thinking so that mental processes are made explicit.

Pupils become increasingly involved as you encourage them to think about the task, ask questions, offer contributions and test ideas.

The oral rehearsal of ideas also provides pupils with a good model, which they can then develop in small groups.

Ensure that this activity is more than 'teacher talk'. Provide pupils with resources to match the display model and encourage them to give commentaries of their own.

Design tasks for pupils to tackle, after your modelling session, that will make talk essential.

## Classifying

Classifying is a task well suited to the thinking processes that everyone uses naturally to organise information and ideas.

A typical classification task may involve a card sort. Pupils work together to sort cards into groups with common characteristics that establish criteria for classification. Being asked to consider and justify their criteria helps pupils to develop their skills and understanding.

The key part of designing a good classification task is the initial choice of cards that will provide a sufficiently high challenge. A common mistake when running a classification task is to intervene too soon and over-direct the pupils.

## Matching

Matching different forms of representation often involves carefully selected cards and a common lesson design. In this instance, pupils are asked to match cards that are equivalent in some way.

This kind of activity can give pupils important mental images, at the same time as offering the chance to confront misconceptions.

## Place value and ordering

You can use these techniques and exercises when planning lesson sequences in place value and ordering.

In primary school pupils will have worked extensively with the image of the number line. As their knowledge and experience of mathematics extends they will welcome the place-value chart as a most flexible and useful image. It will help extend their ability to deal with large and small numbers.

The ability to multiply and divide by any integer power of 10 and to start writing numbers in standard form depends on a secure understanding of place value. This understanding is fundamental in manipulating large and small numbers, both mentally and in written form.

[Multiply and divide integers and decimals by powers of 10](#) provides contexts in which pupils should develop mental processes in place value.

### Multiplying and dividing numbers by powers of 10

Use positive integer powers of 10 and refer to prior knowledge of the way in which a division fact can be derived from a known multiplication fact.

Include the vocabulary of multiplication and division as inverse operations.

Begin with multiplying and dividing by 10, 100, etc. For example:

$$5 \text{ \&CenterDot; } 32 \times 10 = 53 \text{ \&CenterDot; } 2$$

$$53 \text{ \&CenterDot; } 2 \text{ \&div; } 10 = 5 \text{ \&CenterDot; } 32$$

$6 \times 95 = 695$	$695 \div 95 = 6$
$4 \times 78 = 4780$	$4780 \div 78 = 4$

Extend the same approach and understanding to multiplying and dividing by 0.1, 0.01, etc. For example:

$6 \times 0.1 = 0.6$	$0.6 \div 0.1 = 6$
$7 \times 0.01 = 0.07$	$0.07 \div 0.01 = 7$

### Understanding the effect of multiplying and dividing by numbers between 0 and 1

Use powers of 10 as the multiplier to help pupils recognise the logic of the emerging pattern.

Pupils should understand that multiplying by any number between 0 and 1 makes the number smaller. For example:

$4.2 \times 100 = 420$	[makes bigger]
$4.2 \times 10 = 42$	[makes bigger]
$4.2 \times 1 = 4.2$	[stays the same]
$4.2 \times 0.1 = 0.42$	[makes smaller]
$4.2 \times 0.01 = 0.042$	[makes smaller]

Pupils should also understand that dividing by any number between 0 and 1 makes the number bigger. For example:

$4.2 \div 0.01 = 420$	[makes bigger]
$4.2 \div 0.1 = 42$	[makes bigger]
$4.2 \div 1 = 4.2$	[stays the same]

$4.2 \div 10 = 0.42$	[makes smaller]
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$4.2 \div 100 = 0.042$	[makes smaller]
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## Multiplying and dividing decimals by any number between 0 and 1

Use mental calculations with whole numbers and adjust, using knowledge of the effect of multiplying or dividing by numbers between 0 and 1.

Multiplying:

$$31 \times 0.4$$

$$31 \times 4 = 124$$

10 times smaller is 12.4

$$0.25 \times 0.03$$

$$0.25 \times 3 = 0.75$$

100 times smaller is 0.0075

Dividing:

$$81 \div 0.3$$

$$81 \div 3 = 27$$

10 times bigger is 270

$$0.24 \div 0.06$$

$$0.24 \div 6 = 0.04$$

100 times bigger is 4

Alternatively, use the definition of fractions as division and knowledge of equivalent fractions.

Dividing:

$$81 \div 0.3 = 27$$

$$\frac{81}{0.3} = \frac{810}{3} = \frac{270}{1} = 270$$

$$0.24 \div 0.06$$

$$\frac{0.24}{0.06} = \frac{24}{6} = \frac{4}{1} = 4$$

### **Beginning to write numbers in standard form**

Use movements on a place-value grid. Relate numbers back to the 'baseline' of unit digits and describe movements in terms of multiplication, first by multiples of 10 then by powers of 10.

Ask pupils to write the following types of numbers in standard form:

- large numbers, for example  
 $235.7 = 2.357 \times 10^2$
- small numbers, such as  
 $0.00092 = 9.2 \times 10^{-4}$

Pupils should also be able to order numbers in standard form, for example:

- $6.92 \times 10^{-4}$   
,
- $2.5 \times 10^{-2}$   
,
- $3.7 \times 10^2$

# Place value and ordering: Activities

Use these activities to strengthen your pupils' understanding of place value and ordering.

## Using a place-value chart: Activities

Modelling different ways in which pupils can use a place-value chart is fundamental to the tasks described here.

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

Illustrate the multiplicative relationships within each column of the chart by discussing the multiplication that takes you from one column entry to another, for example:

$0.08 \times 10 = 0.8$   
, a step up one row

$0.008 \times 100 = 0.8$   
, a step up two rows.

Link the steps to the powers of ten and establish that multiplying by  $10^3$  is the same as multiplying by 1000 and is equivalent to a step up three rows.

Confirm that moving down within a column can be considered as division or multiplication, for example:

$8 \div 100 = 0.08$   
, a step down two rows

$8 \times 0.01 = 0.08$   
, a step down two rows

$8 \times 10^{-2} = 0.08$   
, a step down two rows.

### Hide and reveal

These tasks engage pupils in the same movements around the chart.

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

Use blank strips to *hide* rows of numbers and ask pupils to explain how they know what numbers are under the strip.

Alternatively, *reveal* only one row of the chart and ask pupils to complete the rows above and below it. As a more challenging task, ask for the row three steps above or below.

Windows in the chart can be used as the basis of a task in which pupils are asked to create the surrounding entries or even the whole chart, possibly using a spreadsheet.

### 2-by-2 window on place-value chart

0.05	0.06
0.005	0.006

This chart can reinforce pupils' understanding of the effect of multiplying by numbers smaller than 1.

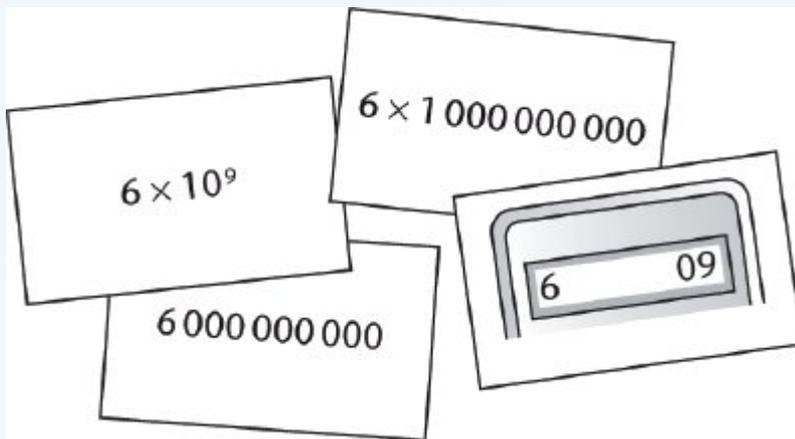
It also allows discussion of the fact that the steps can be reversed in two ways:

- by using the same operation (multiplication) with an inverse operator  $10^{-4}$  or 0.0001
- by using the same number ( $10^4$  or 10 000) and an inverse operation, division.

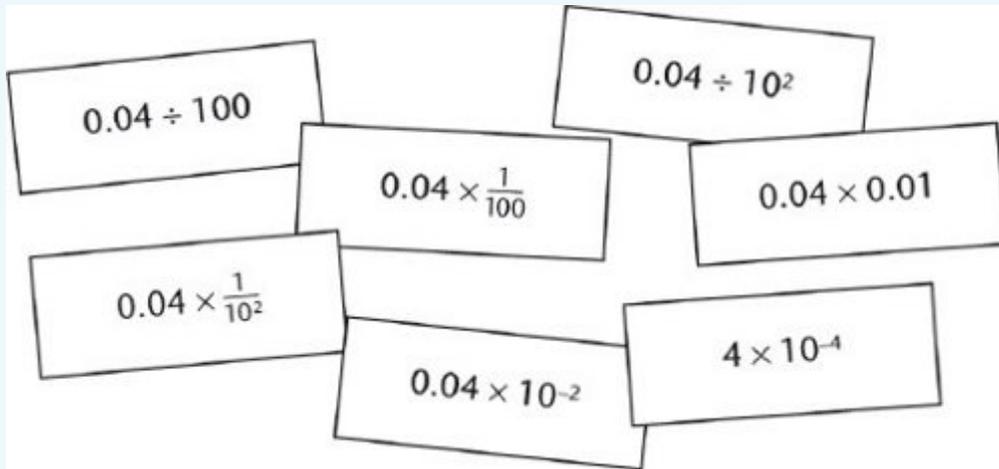
### Matching: Different representations

Matching different forms of representation can provide pupils with the chance to confront misconceptions. For example, pupils could be asked to match cards that show numbers with cards that show calculations and calculator displays in standard form.

For example:



Alternatively, the cards might only show the same calculation written in different ways.

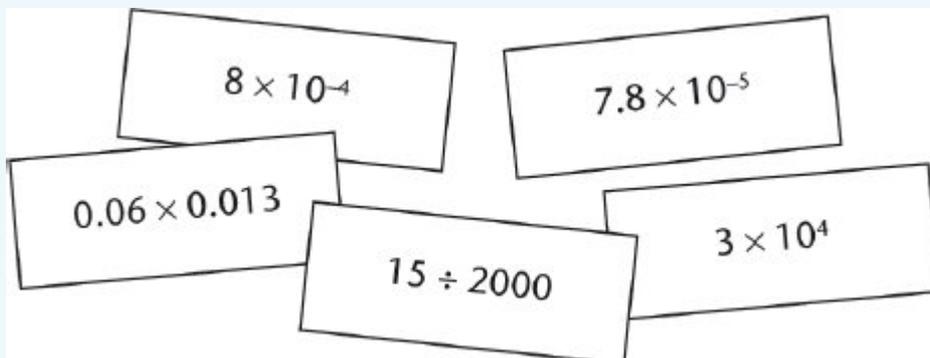


Note that in this task, numbers are written in standard form without necessarily using the term 'standard form'. The introduction to standard form itself is described in 'Writing numbers in standard form: Activities'.

### Ordering numbers and calculations

Ordering without using a calculator focuses attention on place value.

Write multiplication and division calculations (including numbers in standard form) on cards and ask pupils which cards they can put in sequence, in ascending or descending order, without calculating the values of the numbers.



Using cards to match and sequence encourages pupils to discuss their calculations and justify their solutions. Using different sets of cards for different groups of pupils provides a straightforward way of differentiating.

Note that in this task, numbers are written in standard form without necessarily using the term 'standard form'. The introduction to standard form itself is described in 'Writing numbers in standard form: Activities'.

### Fill in the ...

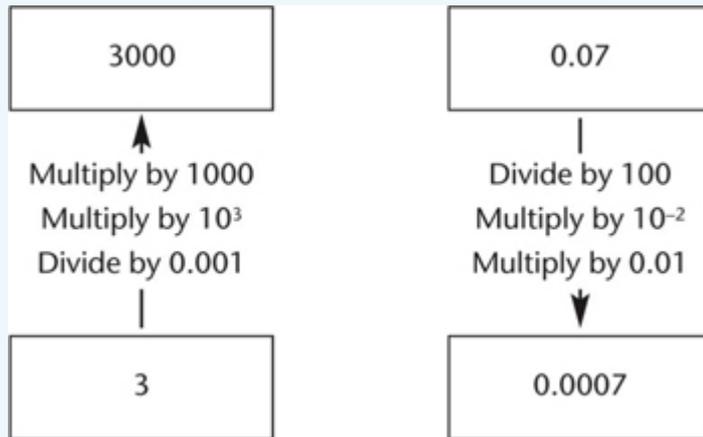
'Fill in the missing numbers' is an activity in which pupils write in the numbers on cards with statements such as:

? multiplied by  $10^3$  is ?

? divided by 0.1 is ?

Pupils work in pairs, using a collection of cards with blank entries. They refer to copies of a place-value chart to support the task. This is about using the structure of the chart, not about calculating.

'Fill in the missing operations' is the reverse of the above task. Pupils continue to work in pairs and to use the structure of the place-value chart. They should describe the operation needed to get from the start number to the second number in as many ways as they can. The task is simplified if only multiplication is used.



The aim is for pupils to become confident in making multiplicative statements, linking pairs of numbers from the same column in the place-value chart. Their increased confidence will be evidence of a greater understanding of the nature of the base-10 number system.

## Writing numbers in standard form: Activities

Modelling how to write numbers in standard form can also begin with the place-value chart.

### Modelling: Using the place-value chart

Identify the 'units' digit line as the baseline of the whole place-value chart. All other lines can be generated from this one by using multiplication to step up or down from it. Ask pupils to describe the steps.

Discuss the numbers in the 'thousands' row, for example:

$$4000 = 4 \times 10^3$$

$$5000 = 5 \times 10^3$$

Generalise to show that all the numbers in the 'thousands' row will be represented by a units digit multiplied by  $10^3$ :

$$n \times 10^3$$

Give pupils copies of one column from the place-value chart and ask them to annotate each of the entries.

Encourage pupils to speculate how to write 4500 in this way. Referring to the place-value chart, ask:

- Where would you start from?
- Where would you place the resulting number?

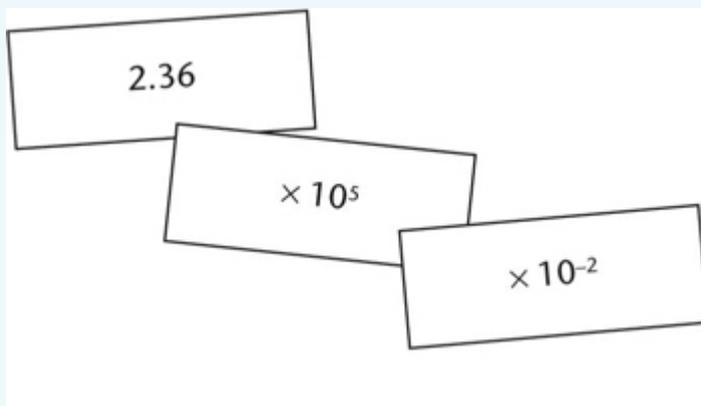
Encourage pupils to imagine a number line running across each row. Explain that this helps them to think about 4.5 and 4500 relative to the numbers shown on the chart.

For example, establish that 4.32 is between 4 and 5, closer to 4 than 5. Ask pupils to perform the same operation for 4.32 as above, deciding which multiplication will take it into the row for thousands. Ask:

What is the equation and where would the number lie on the place-value chart?

$$4.32 \times 10^3 = 4320$$

- Show pupils cards with numbers between 1 and 10 and with  $\times 10$  to a suitable index. Ask them to use the place-value chart to identify the resulting number.



## Multiplication and division with decimals: Activities

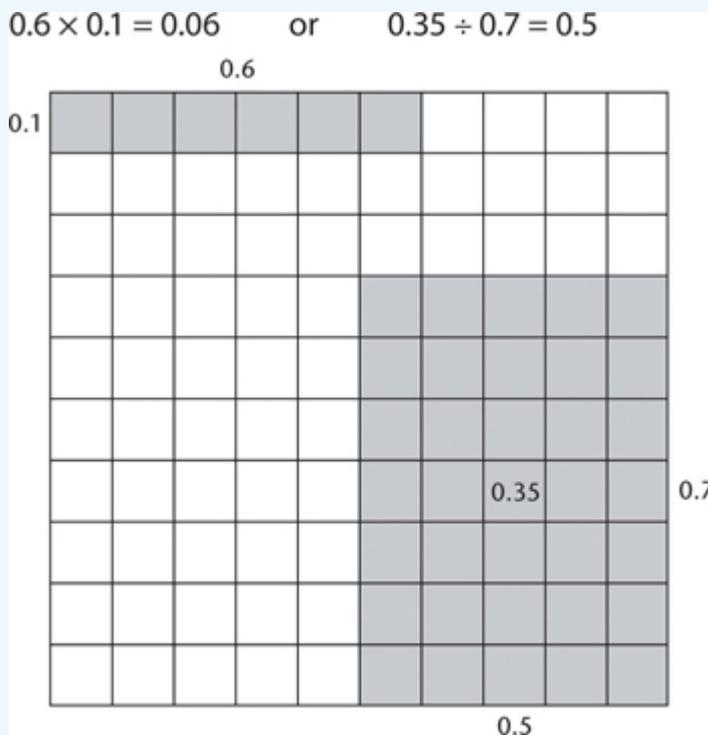
The area model is a useful starting point from which to develop understanding of multiplying and dividing by numbers smaller than 1, especially to show how a number can be partitioned into tenths. It provides a powerful visual image for pupils and can aid mental calculation.

### Modelling: Diagrammatic explanation

Use a diagrammatic explanation such as a rectangular array to show multiplication and division.

Establish first that the unit lengths are divided into tenths and the unit area is divided into hundredths.

For example:



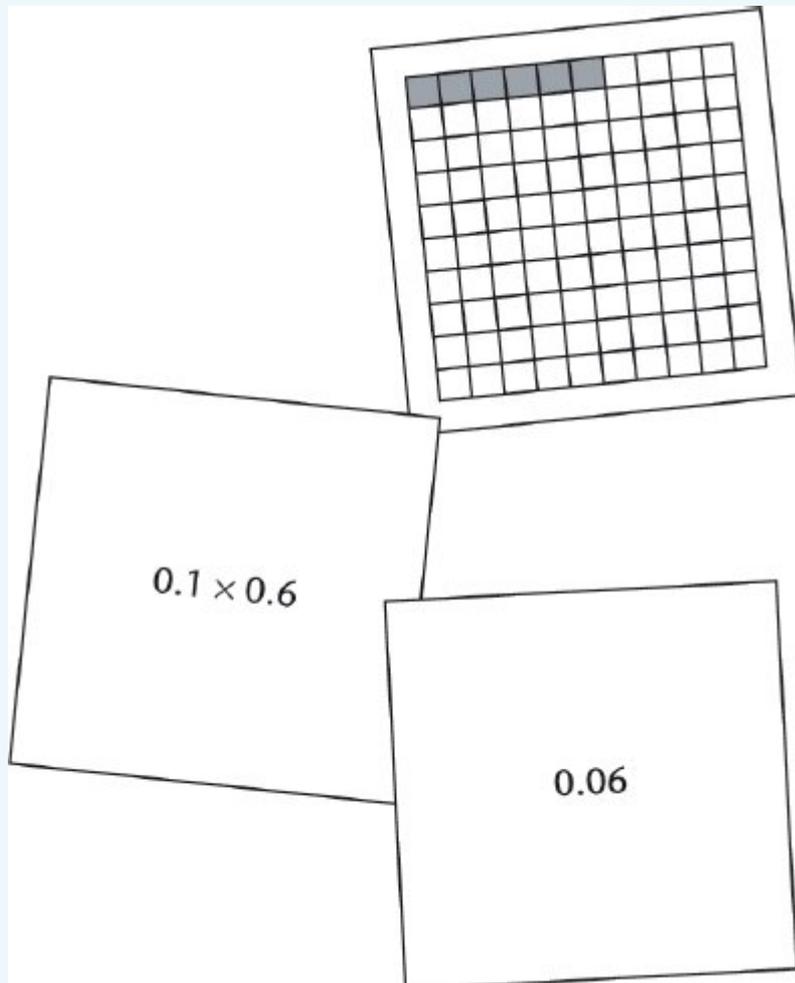
### Matching activities

Ask pupils to match arrays and calculations and then to sketch their own arrays for simple decimal multiplication and division calculations.

Challenge pupils to explain how they could extend this array for larger and smaller numbers.

Matching a calculation with an answer and a grid representation will consolidate pupils' understanding of this image.

For example, ask pupils to match a set from a mixture of cards including examples such as:



This idea is also illustrated in [Multiply and divide integers and decimals by powers of 10](#).

### Find as many ways as possible

Asking pupils to find as many ways as possible helps them to:

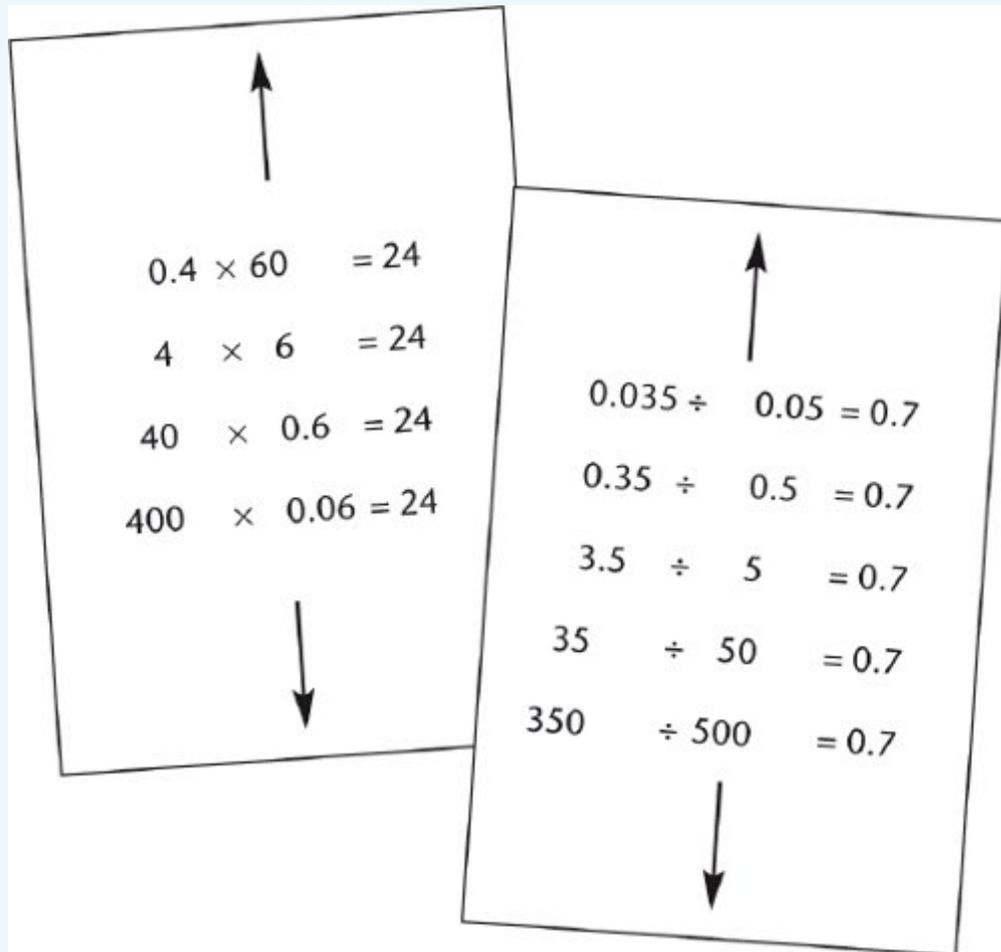
- appreciate that a calculation can be written in several ways
- demonstrate that one calculation is equivalent to another
- discuss which transformation is the most efficient for each calculation.

Give pupils a calculation and ask them to extend and develop it to find equivalent calculations.

Start from a simple known fact, for example,  
 $4 \times 6 = 24$

. Ask questions to lead pupils to consider the effect of making the first number 100 times smaller while the answer remains the same.

What effect does this have on the magnitude of the second number in the calculation?



Encourage pupils to make generalisations.

Spend some time exploring the different equivalent calculations that are obtained for multiplication and division.

Although pupils may use a calculator to check their answers, they should justify the equivalence through the logic of the calculation.

Node information

Attachments Zip:

 [163aadda15ad46f41c9d16b6a081ad7b.zip](#)

## File Attachments

- [find\\_as\\_many\\_ways.jpg](#) ( jpg 20 KB )
- [place\\_value\\_chart.jpg](#) ( jpg 26 KB )
- [modelling\\_diagrammatic.jpg](#) ( jpg 56 KB )
- [matching\\_activities.jpg](#) ( jpg 21 KB )

# Rounding

You can prepare your lessons and help pupils comprehend rounding with the aid of this material.

Mathematics is sometimes regarded as a subject in which all answers must be either right or wrong, but there are many occasions in everyday life when an approximate answer is appropriate. For example, the newspaper headline: '75 000 fans watch Manchester United beat Arsenal' does not mean that exactly 75 000 people attended the match. One of two things may have happened. Either people entering the grounds were counted as they passed through the gate, and the exact number was rounded to the nearest 1000 or 5000, or someone estimated the number. Either case is good enough for the newspaper headline.

Similarly, the value of  $\pi$  is often taken as 3.14 because this is sufficiently accurate for the purpose, even though  $\pi$ , an irrational number, cannot be written precisely as a decimal.

[Round numbers, including to a given number of decimal places](#) provides contexts in which pupils should develop mental processes in this area.

## Introducing rounding to pupils

In calculations, it is often difficult for pupils who have been accustomed to giving 'right answers' to understand that it is not always either possible or necessary to be exact. They need time to develop the idea of a number being 'suitable for the purpose'.

Estimating an answer before starting a calculation is important as it can reveal subsequent errors, particularly when a calculator is used.

For example, before they start to multiply

$$3.7 \times 0.83$$

, it is useful for pupils to recognise that the answer will be less than 3.7 because they are multiplying by a number less than 1.

Rounding the numbers to

$$4 \times 0.8$$

gives 3.2, which is a sufficiently close approximation and can be calculated mentally.

The accurate calculated answer, 3.071, may be rounded to an appropriate degree of accuracy.

When solving any problem, pupils need to know the stages at which it is appropriate to round the numbers and the effect this will have on the result.

### **Rounding to whole numbers and specified numbers of decimal places**

Work the problem both ways, considering the values that a number might have taken before rounding, i.e. 'unrounding'.

Establish criteria for rounding and when a 'trailing zero' is required.

Here are some examples of decimals to the nearest whole number or to one or two decimal places:

$$4.48 = 4 \text{ (to nearest whole number)}$$

$$4.48 = 4.5 \text{ (to 1 decimal place)}$$

$$4.97 = 5 \text{ (to nearest whole number)}$$

$$4.97 = 5.0 \text{ (to 1 decimal place)}$$

$$7.499 = 7 \text{ (to nearest whole number)}$$

$$7.499 = 7.5 \text{ (to 1 decimal place)}$$

$$7.499 = 7.50 \text{ (to 2 decimal places)}$$

## Rounding to specified numbers of significant figures

This way of rounding is most helpful when dealing with very large and very small numbers. It is a precursor to standard form.

For large numbers the result can look the same as rounding 'to the nearest 10 000', for example.

For very small numbers it allows rounding without everything shrinking to 0.

Draw attention to the roles of zero:

- 'leading zeros' as in 0.000 003 76
- 'trailing zeros' as in 458 000
- 'significant zeros' as in 670 006.

In all cases zero is a place-holder maintaining the value of the other digits and therefore the size of the number. In the first two cases the zeros can be established by multiplying or dividing by a power of ten (as in standard form) but significant zeros cannot be repositioned in this way.

Here are some examples of rounding numbers to a given number of significant figures:

$$3768 = 4000 \text{ (to 1 significant figure)}$$

$$3768 = 3800 \text{ (to 2 significant figures)}$$

$$3768 = 3770 \text{ (to 3 significant figures)}$$

$$0.002\ 61 = 0.003 \text{ (to 1 significant figure)}$$

$$0.002\ 61 = 0.0026 \text{ (to 2 significant figures)}$$

$$0.0296 = 0.030 \text{ (to 2 significant figures)}$$

$$0.005\ 04 = 0.0050 \text{ (to 2 significant figures)}$$

$$2\ 083\ 452 = 2\ 100\ 000 \text{ (to 2 significant figures)}$$

### Understanding upper and lower bounds for discrete and continuous data

Draw attention to the fact that rounding effectively maps an interval of numbers onto a single value.

Here is an example using discrete data:

The population of London = 9 million people (to the nearest million)

Then the population must be at least 8 500 000 and at most 9 499 999.

$$8\,500\,000 \leq (\text{population of London}) \leq 9\,499\,999$$

Here is an example using continuous data:

The distance from Exeter to Plymouth = 62 km (to the nearest km)

Then the distance is 61.5 km or further but not as great as 62.5 km.

$$61.5 \text{ km} \leq (\text{Exeter to Plymouth}) < 62.5 \text{ km}$$

## Rounding: Activities

Use these activities to strengthen your pupils' understanding of rounding.

### Four in a row

'Four in a row' gives pupils practice in rounding to one decimal place. This game is a good precursor to further activities.

Pupils play the game in groups, using counters in two colours, two identical sets of cards numbered 6, 7, 9, 11, 13 and 14, and a 5 by 5 grid. They write 25 numbers, all in the range 0.1 to 1.9 and with one place of decimals, randomly on one 5 by 5 grid, as a baseboard.

Pupils take turns to choose two cards (one from each set) and divide one number by the other, using a calculator as appropriate. They round the answer to one decimal place and then place a counter of their own colour on that number on the baseboard.

The aim of the game is to get four counters in a row.

### Before or after?

'Before or after?' is a task that involves pupils in considering the different effects of rounding numbers before or after a calculation.

Pupils work in pairs, using a pile of cards showing four-digit numbers with three decimal places, such as 4.652, 3.894, 2.453, 8.264, 0.675 and 7.329.

They each choose a card and put them together to form a multiplication calculation, for example:

$$\boxed{4.652} \times \boxed{3.894}$$

First they predict the effect of rounding before the calculation. Ask:

- Will the result be smaller or larger than the result of rounding after the calculation?
- Is it possible to say?

Then they test their predictions by rounding each number to two decimal places and multiplying them together (with a calculator). They round the result to two decimal places. For example:

$$4.65 \times 3.89 = 18.09$$

Finally, they multiply the original numbers and round the result to two decimal places:

$$4.652 \times 3.894 = 18.11$$

Encourage pupils to identify and discuss why rounding before computation gives a different answer.

This task can be adapted to use different operations.

Extend the activity with questions such as:

- Which of the numbers you used gave the greatest difference? Why?
- Which gave the smallest difference? Why?

## File Attachments

- [rounding\\_activities.jpg](#) ( jpg 3 KB )

# Fractions, decimals, percentages, ratio and proportion

You can prepare your lessons and help pupils comprehend fractions, decimals, percentages, ratio and proportion with the aid of this material.

This material addresses:

- understanding and using equivalences between fractions, decimals and percentages
- using proportional reasoning to solve a problem.

After calculation, the application of proportional reasoning is the most important aspect of elementary number work. Proportionality underlies key aspects of number, algebra, geometry and statistics.

In order to make progress through levels 6, 7 and 8 in the Number section of the National Curriculum, pupils must recognise which number to consider as 100% or a whole. This enables them to reverse proportional change and to calculate repeated proportional change.

In many problems, making the appropriate choice between fractions, decimals, percentages or ratio will be crucial. Pupils can only make such a choice if they have a sound understanding of equivalence.

Developing mental mathematics from level 5, in this context, means securing a flexible approach to a problem, underpinned by confidence with equivalence and multiplicative strategies.

## Understanding and using the equivalences between fractions, decimals and percentages

Use the following approaches and goals when teaching pupils to understand and use the equivalences between fractions, decimals and percentages.

Pupils should already have some knowledge of the equivalences between different representations of fractions, decimals and percentages.

As pupils progress beyond level 5 it is important they recognise that the choice of form can affect the ease and efficiency with which a calculation can be performed, particularly mentally.

Pupils need to become confident in spotting appropriate forms and converting between them. This is usually a mental process, even when the subsequent calculation is performed on paper or by calculator.

Build learners' confidence and fluency by creating chains of equivalences, using striking visual arrangements to draw attention to the structures and patterns.

### Goals

Pupils should know and use the equivalence between fractions, decimals, percentage and ratio.

In this table are examples of key concepts pupils should know and use:

**Know and use the equivalence between fractions, decimals, percentage and ratio:**

Know and use the equivalence between fractions, decimals, percentage and ratio:	
Know that $\frac{3}{5}$ , $\frac{3}{5}$ and $3 \div 5$ all means the same	Relate fractions to division
Know that $7 \times \frac{1}{3}$ , $7 \div 3$ , $\frac{1}{3}$ of 7, $\frac{7}{3}$ and $2\frac{1}{3}$	Interpret different meanings of fractions are equivalent
$0.365 = \frac{365}{1000} = \frac{73}{200}$ , $0.365 = 36.5\%$	Convert terminating decimals to fractions or percentages
$\frac{1111}{36911}$	Convert common recurring decimals to fractions
$\frac{1}{3} = 0.3333 = 0.\dot{3}$ , $\frac{2}{9} = 0.2222 = 0.\dot{2}$ , $\frac{7}{6} = 1.1666 = 1.1\dot{6}$	For simple fractions with recurring decimal equivalents, convert between the fraction and decimal forms
$\frac{1}{3}$ , $\frac{a}{3a}$ , $\frac{2(a+b)}{6(a+b)}$	Simplify or find equivalent algebraic fractions

# Understanding and using the equivalences between fractions, decimals and percentages: Activities

You can use these activities and approaches when teaching pupils to understand and use the equivalences between fractions, decimals and percentages.

[Fractions, decimals, percentages, ratio and proportion](#) and [Recall of fraction, decimal and percentage facts](#) provide contexts in which pupils can develop mental processes in fractions, decimals, percentages, ratio and proportion.

## Within and between

'Within and between' is a task in which pupils use arrays to simplify or find equivalent fractions or ratios.

For example:

	Equivalent fractions		
Numerator	0.7	7	35
Denominator	?	?	150

$\times \frac{30}{7}$

$\times \frac{1}{5}$

The diagram shows a table with three columns of equivalent fractions. The first column has a numerator of 0.7 and a denominator of ?. The second column has a numerator of 7 and a denominator of ?. The third column has a numerator of 35 and a denominator of 150. An arrow points from the second column to the third column with the multiplier  $\times \frac{30}{7}$ . Another arrow points from the first column to the second column with the multiplier  $\times \frac{1}{5}$ .

Pupils can simplify the single multipliers as shown. For example, to get from 7 to 35, use or to find the number, 30, to write in the cell.

They should notice that the link between the numerator and denominator is the same for every pair. The link between a pair of numerators (and their equivalent denominators) is different for different pairs.

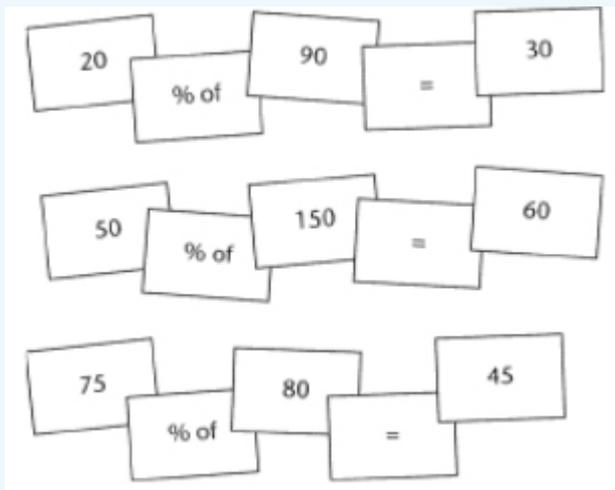
When working with ratio the array would look similar.

2	:	?
14	:	35
?	:	85

Observation and explanation of this 'within and between' relationship for a proportional set is very important. The emphasis is on noting down the calculations, for example,  $150 \times \frac{30}{7}$ , rather than the actual computation.

## Sorting

Sorting activities in the form of a puzzle can help pupils to find different equivalences.



Pupils work in pairs to sort a set of number cards and rearrange them to make three correct calculations.

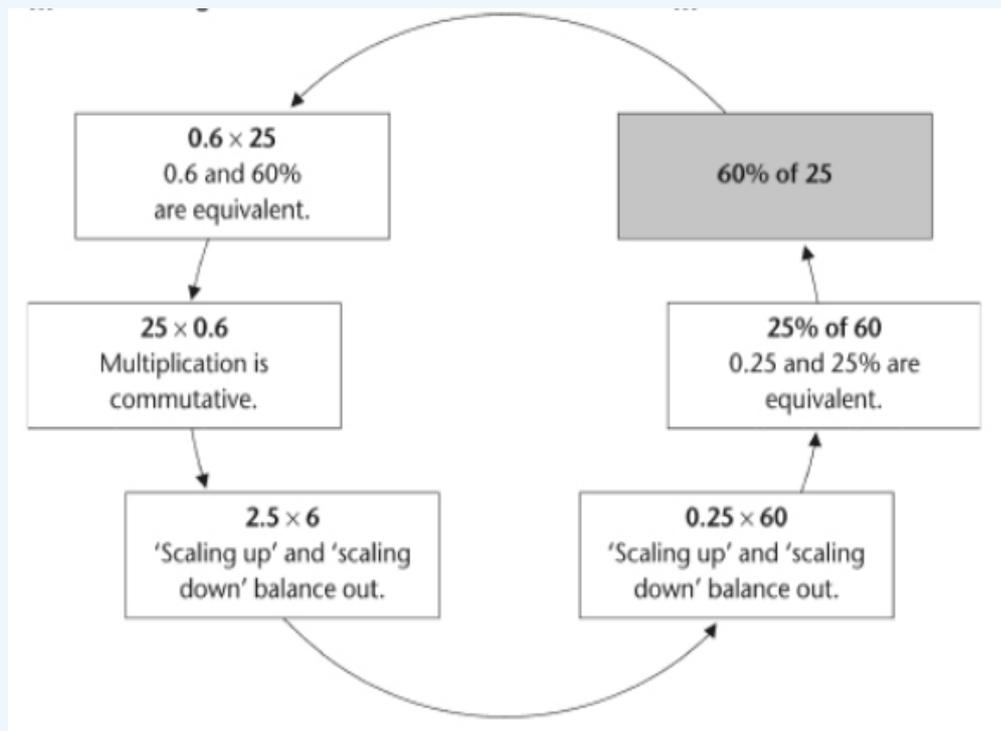
This activity requires pupils to calculate mentally while sorting and re-sorting the cards until they make all the calculations correct.

The activity may be varied by using fractions or decimals rather than percentages.

### Percentages: a chain of reasoning

'Percentages: a chain of reasoning' is a task that encourages pupils to think flexibly about a calculation. It also encourages pupils to give reasons and justify steps they are taking.

In this example, the chain ultimately shows that 60% of 25 is the same as 25% of 60.



### Clouding the picture

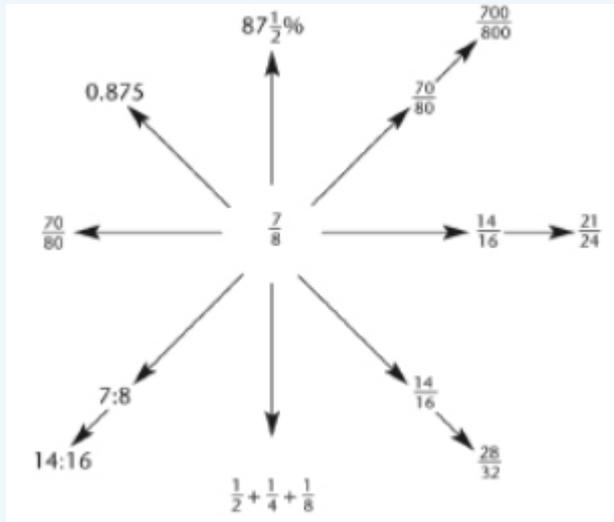
Clouding the picture is a technique to enable pupils to identify other related facts by using equivalence.

For example, ask pupils to complicate the central fraction [mathml:] in as many ways as they can.

They should:

- start by giving another couple of examples along each branch

- stop after a few examples and try to explain what is happening along the branch (generalising the process)
- start a new branch that does something different to complicate the fraction.



This technique could be modelled by the teacher and then repeated by pupils, using different fractions, decimals, percentages or ratios as the centre numbers. The task can easily be adjusted to match the challenge to the group of pupils.

## Using proportional reasoning to solve a problem

Use these concepts when teaching pupils to use proportional reasoning to solve a problem.

Develop the concept of a single multiplier, using diagrams or tabular arrays to represent the data.

This table shows examples of key concepts pupils should know and use:

Example	Concept
0.625 is equivalent to $\frac{5}{8}$ and 62.5%	Express a decimal as a fraction or a percentage

<p>21% of £3000</p>	<p>Interpret percentage as the operator 'so many hundredths of'</p>
<p><math>\frac{2}{3}</math> of 75</p>	<p>Interpret a fraction as the operator</p>
<p>2 as a percentage of 8 is 25%</p>	<p>Express one number as a percentage of another</p>
<p>5 as a fraction of 3 is <math>\frac{5}{3}</math> or <math>1\frac{2}{3}</math></p>	<p>Express one number as a fraction of another</p>
<p>An increase of 23% is equivalent to a single multiplier of 1.23</p> <p>A decrease of 23% is equivalent to a single multiplier of 0.77</p>	<p>Find the outcome of a given percentage increase or decrease</p>
<p>In a pastry recipe the ratio of fat to flour is 1 : 2.</p> <p>For every 1 g of fat there are 2 g of flour so the weight of the fat is <math>\frac{1}{2}</math> the weight of the flour.</p> <p>In the total weight, 1 g in every 3 g is fat. The proportion of fat in the total weight is <math>\frac{1}{3}</math>.</p>	<p>Understand the relationship between ratio and proportion</p>
<p>After a pay rise of 5%, Dave was paid £5.04</p>	<p>Identify 100% when the increase or decrease has already taken place</p>
<p>The price of a CD is reduced by 12% and then by a further 7%</p> <p><math>\frac{3}{4}</math> of the class are girls and <math>\frac{1}{2}</math> of the boys have their ties on</p>	<p>Solve problems involving repeated proportional change</p>

# Using proportional reasoning to solve a problem: Activities

You can use aligning diagrams and branching diagrams when teaching pupils to understand and use proportional reasoning to solve a problem.

## Aligning diagrams

'Aligning diagrams' can help pupils to organise their thinking around the calculations required to solve a problem. They can help to clarify:

- the layout of the data as it is extracted from the problem
- the proportional relationships and hence the layout of the calculations required.

Using a pair of number lines to illustrate multiplicative relationships is a simple alternative to extracting the data into a two-way table.

### Example 1

Aaron earns £42 a week. He spends 23% of his earnings on CDs.

How much money does he spend on CDs each week?

#### Method 1: Using number lines

Show pupils how to extract the relevant information and display it on a number line.

This diagram leads to the calculation

$$23 \times \frac{42}{100}$$

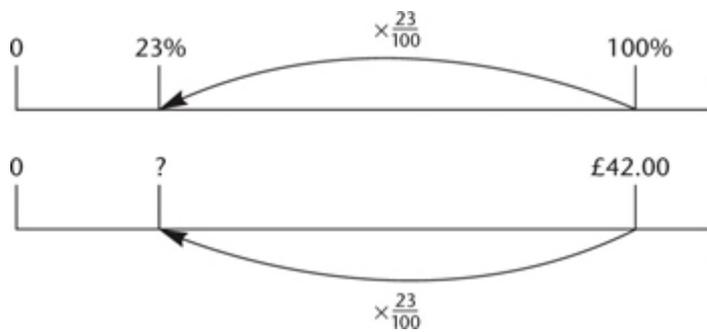
:



This diagram leads to the calculation

$$42 \times \frac{23}{100}$$

:



By performing these steps, the key strategy of finding a single multiplier is developed.

### Method 2: Using a two-way table

The information can also be organised into a two-way table:

Percentage (%)	23	100
Money (£)	?	42.00

The problem can be solved by using a single multiplier. Looking at the relationships:

- from right to left gives  
 $42 \times \frac{23}{100}$
- from top to bottom gives  
 $23 \times \frac{42}{100}$

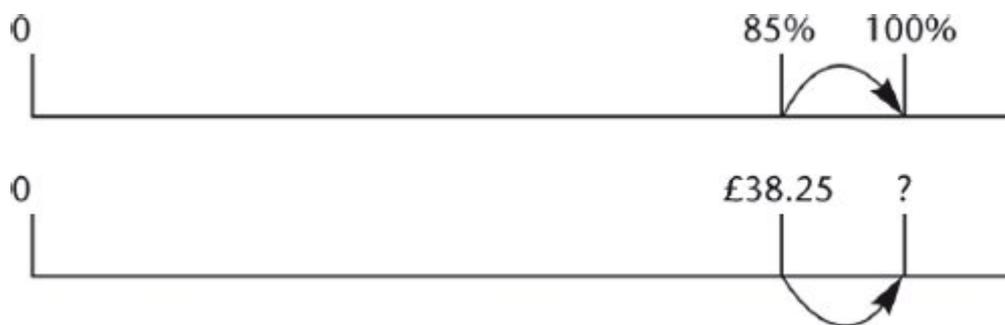
The strength of both these methods is that they need not be changed for more complex problems.

## Example 2

A shop had a sale. All prices were reduced by 15%.

A pair of shoes cost £38.25 in the sale. What price were the shoes before the sale?

### Method 1



### Method 2

Percentage (%)	85	100
Money (£)	?	38.25

These are both good models for solving problems of this type because, when quantities are in proportion, the corresponding ratios within or between these quantities are equal.

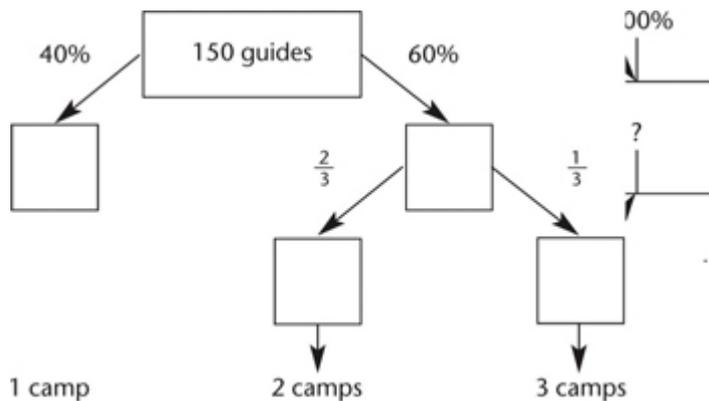
## Follow-up work

Give pupils a variety of problems. Ask them to work in pairs to extract relevant information and then to organise this information on a pair of number lines or in two-way tables. Ask them to identify the single multipliers and resulting calculations necessary to solve the problem.

## Branching diagrams

A branching diagram can model the steps of a problem. For example:

There are 150 guides at a guide camp. For 40% of them, this is their first camp. Two thirds of the remainder have been to one other camp and the others have all been to three camps in total. How many guides have been to three camps?



Pupils could use this type of diagram to organise a variety of problems and then pose additional questions for a partner to solve. This is useful preparation for organising branching diagrams in the context of probability.

Node information

Attachments Zip:

 [335d9c907b6b6b4c9eefc0907d940a08.zip](#)

## File Attachments

- [Alternative number lines for Example 1](#) ( jpg 24 KB )
- [Branching diagram for guide camp example](#) ( jpg 31 KB )
- [Number lines for Example 1](#) ( jpg 22 KB )
- [Number lines for Example 2](#) ( jpg 5 KB )