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Sir Derek Morris
The Morris Review
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Dear Sir Derek

I had not sent you comments previously, but after reading your Interim Assessment Report and attending the recent meeting at the Faculty of Actuaries, it seems worth while my letting you know my views.

I should like to start with an analogy. It is imperfect, but it may be relevant. At the beginning of the nineteenth century doctors still used bloodletting, perhaps with leeches, as the main method of treatment. By the end of that century they knew about bacteria, and understood the benefits of isolation, asepsis and antisepsis, even if they did not yet have the drugs to deal with infectious diseases. At some time during that century there were presumably older doctors, who knew the old ways, at the same time as younger doctors, who had learned about the new ideas. The conflict between them is nicely dramatised in a novel *The Siege of Krishnapur*, by J. G. Farrell, who sets his story in a besieged town during the Indian mutiny, in which there is an outbreak of cholera and there are two doctors who have very different ideas about how to deal with it.

The actuarial profession is in the same state of transition. When I qualified 45 years ago I had learned actuarial mathematics from textbooks that were essentially unchanged from the *Institute of Actuaries Text Book* of 1870. This in itself is no fault; basic geometry has not changed since the time of Euclid, and his *Elements* would still be useable as a basic geometry textbook nowadays; but mathematics as whole has moved on since his day. However, the actuarial textbooks of the 1950s, although they mentioned the necessity for taking cautious margins, and keeping contingency reserves to deal with uncertainties, had no suggestions as to how to measure those uncertainties.

Things have moved on since then. The development of stochastic models of all kinds, in the investment field, in dealing with demographic concepts, and in dealing with general insurance risks, has proceeded. "Collective risk theory" in the general insurance field had indeed been developed, in Scandinavia, well before the 1950s, but I remember my seniors in a large Scottish life office (Standard Life) being very scathing about these new-fangled ideas. Even now, many practical general insurance actuaries are more concerned with other matters than with this mathematically more difficult approach.

The work of financial economists, bringing a rather different approach to the statistical properties of investment variables has both helped and confused the picture. I suggest that there are two

new schools of thought, and there are actuaries in both "camps", who disagree with each other as well as with the "traditionalists". One camp follows what they believe is the financial economists' view, with simple concepts of efficient markets, no arbitrage, and no market inefficiencies, whereby one can, for example, value an option as a defined present value, and hence get back to certainty in valuation. The other camp (in which I fall) accepts the financial economists' theoretical ideas, but would like to place them in a real world that is described by its own stochastic models, and which recognises market inefficiency, and the uncertainty of one's knowledge.

I, jointly with other authors from Heriot-Watt University, have presented two papers to the profession recently describing our approach to option pricing. They are:

Reserving, pricing and hedging for policies with guaranteed annuity options by A. D. Wilkie, H. R. Waters and S. Yang.

Notes on options, hedging, prudential reserves and fair values by A. D. Wilkie, M. P. Owen and H. R. Waters.

The first has already been published in *British Actuarial Journal* (Vol 9); the second will be. I shall add the papers (which are quite long) as pdf files to my submission, which will be sent by email. In these papers, particularly the more recent one, we emphasise the advantages of conducting the investment policy of a life office (or of any option writer) according to a suitable hedging strategy, but also emphasise that this will not necessarily produce exactly the required outcome, because of a variety of factors: discrete hedging, transaction costs, increments that are not normally distributed, our ignorance of the true values of the parameters of the model we assume, and even our ignorance of the true model followed by real markets. We attempt to describe and measure the uncertainties in the process.

The point of these rather technical remarks is to lead in to two observations.

Observation 1

First, in your Interim Assessment Report, in Section 6, you mention, apparently with approval, the view that actuaries need to know more about stochastic modelling and financial economics. Both of these subjects, to be dealt with adequately, require a good knowledge of certain mathematical concepts. Yet in Paragraph 6.4 and Chart 6.1 you show that about one third of new student entrants in 2002 did not have degrees with any mathematical content. It may well be that not many of these entrants will pass all the examinations. It might be useful also to obtain and show the same figures for those who *qualified* in some recent year. I would expect the proportion of non-mathematicians qualifying to be lower than one third.

There is a conflict, which you do not bring out, between the necessity for all actuaries to have an adequate mathematical understanding, especially of the concepts of probability and statistics, and the desire of clients, employers and many in the profession that actuaries should be good communicators, able to explain their assumptions lucidly and so on. These are not necessarily mutually exclusive. A good mathematician may also be quite capable of explaining his concepts to a receptive lay audience. Some, however, may be technically proficient, but poor communicators. However, an actuary who does not have an adequate mathematical understanding may have nothing that he can reliably communicate. The mathematics is an

essential requirement; the communication is a desirable one. It is nice if a piece of domestic equipment both looks good and works well, but one can use an ugly machine that works, and can't use an elegant one that does not work.

Besides the requirement for all actuaries to have a good understanding of the concepts of probability and statistics, some actuaries need to have a very good understanding of certain mathematical techniques, especially nowadays those relating to the important newer subjects of stochastic modelling and financial economics. "Multi-state" stochastic demographic models generally require the *numerical* solution of quite complex differential equations (*analytical* solution is usually too difficult). The techniques are not too difficult to acquire, as I have found, but they are unfamiliar to many actuaries, especially those who have been trained to think that all problems can be solved with no more than an Excel spreadsheet.

Option pricing methods require either the solution of stochastic differential equations, which involves a type of calculus well beyond what used to be taught at school (I do not know how much calculus is now taught at school), or else the simulation of a large number of alternative outcomes. Both techniques were unfamiliar to the traditional actuary, though "Monte Carlo" simulation methods have been widely used for a long time now. But both techniques can be learned by anyone with a basic mathematical knowledge who has the patience to understand them.

In my view, unless the actuarial profession guards its technical competence, and improves it to meet modern needs, it is in danger of losing out to technically competent non-actuaries. In my view it is far better to maintain a high technical standard, even though fewer entrants can achieve that level, than to have large numbers of those who may have passed professional examinations, but have no real understanding of the subject-matter of their profession. This may sound an elitist view. I am all in favour of there being more actuaries who meet the high standards I think are necessary. I see no need, however, to increase quantity at the expense of quality.

Observation 2

Secondly, when there are two, or even three, schools of thought in the profession, the introduction of prescribed standards, whether through an Actuarial Standards Board (however that is formed and controlled) or otherwise, could well lead to the establishment as mandatory standards of methods and concepts that are becoming out of date, and would therefore hamper the development of actuarial thought, ossifying practice rather than bringing it up to date. Any Actuarial Standards Board is likely to consist of some of the more senior members of the profession, who are more likely to adhere to the older ideas. They are also likely to be respected practitioners, who will safely establish what they know that they and their colleague have been doing, rather than researchers and academics, who better understand the new ideas, but have not had many years practising them.

I am not wholly against the establishment of an Actuarial Standards Board. It seems to me right that where traditional methods are appropriate, some control of the methodology, and in particular of the choice of assumptions, should be enforced, so that wholly unreasonable assumptions are not being made by practitioners. But there is a great danger that traditional methods will be made mandatory, or that new methods will be constrained in inappropriate ways.

I can give three examples.

Example 1

First, the mortality assumption prescribed in the Minimum Funding Requirement, which was established in 1996, is "PA(90) rated down 2 years". PA(90) is a mortality table that I constructed myself about 1974, on behalf of the Continuous Mortality Investigation Committee of the professional bodies, of which I was a member for a long time. The table was based first on the mortality experience of pensioners insured by life offices from 1967 to 1970, and then forecast to represent a possible experience for the calendar year 1990. The basis of forecasting was a reduction of 20 years in age for each 20 calendar years forward, so a reduction of two years to the age is equivalent to the forecast made in 1974 applicable to 2030. This might appear have appeared a not unreasonable assumption in 1996 for the average future lifetimes of members of pension schemes, had my forecasts been conformed by the actual experience. But my 1990 forecast level of mortality was reached by about 1980, so the forecasts were already out of date.

I constructed new tables, again for the CMI, based on the experience from 1979 to 1982, one of the "80 series" tables, which were published in 1988. With a colleague, Professor John McCutcheon of Heriot-Watt University, we developed a new forecasting model, which was published in 1990, and was broadly equivalent to a reduction of one year in age for every 10 calendar years. The new 2010 forecast, which was treated as a standard for comparison, was reached in practice by about 1996. Yet again I constructed new tables for the CMI, this time based on the experience for 1991 to 1994, the "92 series" tables, and Professor McCutcheon developed a new forecasting model. Time after time, improvements in mortality rates, particularly for those in their 60s and 70s of age, have run faster than we expected.

Yet the actuaries who prescribed the Minimum Funding Requirement basis chose to use a mortality table constructed 20 years previously, which had been shown to be out of date and had been replaced entirely by the time the MFR was published, and the MFR has not been altered (though it is being abandoned), even though new tables have been published since the MFR was first established. Indeed a further set of new tables is under construction by the CMI, this time based on the experience for 1999 to 2002.

The big improvements in mortality rates are higher ages are now getting lots of publicity. The actuarial profession has been criticised for not foreseeing these improvements. I and the other members of the CMI Committee indeed did not foresee accurately what has happened, but we did keep our colleagues informed as fast as we could (mortality investigations take an inordinately long time, for reasons I shall not go into), and I am afraid that most of our colleagues ignored what we are saying. I fear that an Actuarial Standards Board might be just as backward, if not about mortality (which is currently in the spotlight) then about some other aspect.

To be fair, Technical Memorandum 1, issued by the actuarial profession in 2002, and relating to Statutory Money Purchase Illustrations, does prescribe up-to-date mortality tables (PMA92 and PFA92 as forecast with reference to the specific year of birth of the applicant), so the actuarial establishment is now up to date at least in this respect.

Example 2

These Statutory Money Purchase Illustrations, however, bring me to my second example, the Technical Memorandum just referred to. This states (1.1.1) that:

> The overall aim is to provide illustrations of pension benefits on a broadly consistent basis across various types of money purchase pension provision.
> This is intended to assist individuals to assess in broad terms
> - the adequacy of their pension arrangements
> - the extent to which they need to make further provision
> - the significant uncertainties involved in using *Illustrations* of benefits.

While there are many references in the Memorandum to the uncertainties of making future predictions, and the dangers of these illustrations being taken as predictions, nevertheless the Memorandum prescribes a single illustration, on a defined basis.

While this Memorandum was being drafted I wrote to the relevant Board, suggesting that, instead of showing one figure, it would be better to show a range of possible results. One possible format might be:

> "There is about an even chance of your pension being between X and Y; the odds of it being below X are about 3 to 1 against; the odds of it being above Y are about 3 to 1 against."

This could be re-expressed in probabilities (½, ¼, ¼) or as percentages (50%, 25%, 25%), as well as in betting odds, in the hope that the recipient understood at least one format. However, I was told that the Department of Work and Pensions, which required the illustrations, required a single figure, so the actuaries had to go along with that. My view is that the actuarial profession should have told the DWP that it was not prepared to specify a single figure illustration, which was almost certain to be falsified, and that it would be misleading for providers to have to provide such illustrations. It may not be just actuaries that look for *Certum ex incertis*.

My view is that providers will in due course find that their policyholders treat the forecasts as if they were guarantees, and that we may well have another "misselling" scandal.

Example 3

As a third example I turn to guaranteed annuity options, which have caused life offices a great deal of trouble and considerable losses in recent years. Their effect on Equitable Life is well known, but Lord Penrose has shown that these were not the only problem of the Equitable.

In the 1970s several life offices had introduce unit linked life assurance. Often they included a "maturity guarantee", a guarantee that the unit value at maturity would be not less than some amount, such as the sum of all premiums paid. It was assumed that share prices would always rise, at least over longer periods, so such guarantees would cost nothing. The sharp fall in share prices in 1973-74 showed that this assumption might not be so reasonable. The actuarial profession set up the "Maturity Guarantees Working Party", of which I was a member. We reported in 1980, recommending a method of setting up "contingency reserves" on the basis of

the quantiles from a large number of simulation (the equivalent of the more modern concept of Value-at-Risk, but measured over the whole lifetime of the portfolio of policies), using a simple stochastic model for share prices. This recommendation was widely accepted and was implemented by most of the life offices writing this business, who also stopped offering such guarantees.

The methodology that was accepted for maturity guarantees was equally applicable to guaranteed annuity options. The first of the papers I am attaching, by Wilkie, Waters and Yang or WWY, showed how life offices which had written polices with guaranteed annuity options could, using methodologies already published by 1984, including a later, more complex stochastic model that I published in 1984, have set up reserves to meet the possible costs, and could have increased those reserves during the 1980s and 1990s as interest rates fell and mortality improved.

A useful report by Bolton *et al*, first published in 1997, and republished in *British Actuarial Journal* (Vol 10) in 2004, gave the results of a survey carried out in 1997 into how life offices reserved for the guaranteed annuity options they had written. About half the offices ignored the guarantees entirely. The other half valued them at the higher of the cash value and the annuity value, in effect valuing an option at its "money value" and ignoring the "time value". By 1997 these options were in many cases "in the money" or close to being in the money, so the time value would have been significant.

I still fail to understand why, when actuaries in life offices understood and made provision for maturity guarantees in the early 1980s, they or their successors had forgotten by the 1990s how to make proper provision for guaranteed annuity options. The Guidance Notes issued at that time make at least an oblique reference to the use of simulation methods for such purposes. The actuaries at the supervisory body, which at that time was in effect the Government Actuary's Department, knew about the earlier problem and how it had been solved at the time. Why did they not recognise a very similar problem, and ask the life offices it to be dealt with in the same way? I do not know.

Of course, nowadays one can tackle guaranteed annuity options also by option pricing methods, as we show in WWY. But life offices were not doing that either.

One can speculate: if one of the actuaries who had made no provision for the option value of the guarantees of his office had been brought before a disciplinary tribunal of the profession, on the grounds that he had failed to recognise a liability that clearly existed, would it have been a reasonable defence that half the other actuaries in the same position were doing the same?

Any disciplinary tribunal, however appointed, whether associated in some way with the professional bodies or not, is going to include some more senior actuaries. It may include lay members, as the present tribunals do. But if those actuaries are typical of their professional colleagues, they will have standards and knowledge similar to that of the rest of the profession. Indeed, anyone who is seen as a revolutionary or an outsider or who has non-standard views is most unlikely to be appointed to such a tribunal. But if the mass of the profession does not appreciate how a new problem facing them should be solved, how can a disciplinary body help? Of course it can deal with obvious malfeasance, fraud, criminal activities, failure to observe laid down procedures, and with a failure even to get traditional methods right. But I do not see that it

can help in any way to move the profession from an older paradigm to a newer one. That is probably not what it is meant to do, but this limitation should be recognised.

Calculation 1

In your Interim Assessment Report, in paragraph 10 of the Executive Summary, you comment that "Actuaries have been criticised … [for] failing to allow adequately for the persistently downward path of inflation and interest rates in the 1990s". I thought that it would be interesting to use my stochastic asset model, as published in 1984, to discover what I would have said, had I been asked to make forecasts in, say January 1990, about the progress of economic and investment variables over the next 10 to 15 years. I did not in fact make such forecasts, but I had the methodology (and the computer programmes) readily available to do so, so I am not arguing from hindsight.

My model at that time covered price inflation, share dividends and dividend yields, and hence share prices, and long-term interest rates. I start with inflation. My model assumes a first order autoregressive model for the "force of inflation" (the continuous rate of increase), and in the 1984 model I assumed a mean continuous rate of increase of 0.05 (or 5.13% per annum). The outcome depends on the starting conditions, and for the 10,000 simulations carried out I assumed the conditions as at 31 December 1989. The Retail Prices Index in December 1989 (based on 100 in January 1987) was 118.8; in December 1999 it was 167.3 and in December 2004 it was 189.9. The mean value of my forecast of the value of the RPI for December 1999 was 216.1, and the median was 203.8, both well above the actual outturn. But my annual standard deviation is quite large (0.05) and the autoregressive feature (parameter 0.6) amplifies any trend. The actual outcome corresponds with a percentile of about 29%, so I would have said in 1989 that the chance of the RPI being below 167.3 by December 1999 was about 29%. This is not so unlikely that it should have been ignored. The range of my 10,000 simulations was from 50.6 to 689.5, but these extreme outcomes could be considered so remote as to be ignored.

My forecasts for the value of the RPI in December 2004, still based on December 1989, show a mean of 289.1, a median of 262.3, and a probability of about 24% that the value would be lower the 189.9 it actually achieved.

Now, after 15 years of lower inflation, I would use a mean rate closer to 0.025, or about 2.5% per year. Many people seem confident that inflation will stay low indefinitely. But now perhaps the risks are either of inflation rising again, or of prices falling noticeably. Either would have significant economic consequences.

I now turn to share prices. The value of the FT-Actuaries (now FTSE-Actuaries) All-share Index on 31 December 1989 was 1204.70. The gross dividend yield was 4.24%, implying a dividend index of 51.08. My model forecasts dividend yields with an autoregressive series, and dividends as a type of random walk superimposed on a lagged version of inflation. From these I derive a value for the share price index. My mean forecast for 1999 for the share price index was 2500.6, and the median was 2228.6. The actual outcome was 3242.06, which is at about the 80% probability point, so not implausible. The actual figure for December 2004 was 2410.75, which is at about the 39% point, so not at all implausible.

However, it is interesting to see how these figures decompose into dividend and dividend yield. The (gross) dividend index in December 1989 was 51.08. The actual figure for December 1999 (grossed up by dividing the now current "actual yield" by 0.9) was 76.37, which was at the 29% position in the forecasts made in 1989, so rather below an average change. But since inflation was a similar amount below its average, this is not surprising. The (gross) dividend yield in December 1989 was 4.24%. The December 1999 (grossed up) figure was 2.36%, at about the 2% probability, so fairly unlikely, viewed from 1989.

By December 2004 the dividend index had risen a little to 81.70, at about the 23% point, so still rather below what might have been hoped in 1989. The dividend yield had risen to 3.39%, at about the 16% probability level, so still below the 1989 expected position. Thus in the period from 1989 to 1999 dividends rose less than expected, and dividend yields by much more. By 2004 the position was similar, but less extreme.

My stochastic model as published in 1984 included a model for long term fixed interest yields. The model allows for a quite slow response to inflation, with an autoregressive "real yield" superimposed. The yield on the FT-Actuaries Irredeemables index on 31 December 1989 was 9.66%. By 31 December 1999 it was 4.89%, at about the 2 in 1,000 point (0.2%). In 1989, following my model, I would have thought that the actual fall in yields was very unlikely. By 2004 the actual yield was 4.45%, again at about the 2 in 1,000 probability level.

My 1984 model did not include short-term interest rates, which I did not include until the 1995 version. But it is possible to plug the parameters of the 1995 model into the 1984 one. Bank base rate, which is my measure of short-term rates (perhaps not the best nowadays, but there is very long history of it), was 15% in December 1989. The December 1999 figure of 5½% was at the 14% probability level, so much less unlikely than the lower rate for long-term stocks. By December 2004 base rate was 4¾%, at about the 10% level, so not implausible.

Therefore I conclude this section by suggesting that criticism of the actuarial profession for failing to forecast the fall in interest rates during the 1990s is misguided. My numerical evidence supports this.

One can carry out the same forecasting exercise using the starting conditions as at 31 December 1999. I restrict myself to share prices. The share price index in December 1999 was 3242.06. The forecasts for five years ahead at that time give a mean of 2570.5 and a median of 2434.2. By December 2004 the index was 2410.75, at about the 49% probability point, so very near the mid point of the distribution. In December 1999 the chance of the index being at a lower level in five years time was over 80%. This was almost entirely because dividend yields were very low in 1999, and P/E ratios were also very high. Any reasonable observer who looked at all at these "fundamentals" would have said that share prices were far too high in 1999. I am surprised by the statement made by you in paragraph 10 that: "Actuaries have been criticised … [for] failing to allow adequately for the subsequent precipitate fall in the stock market". I wonder what evidence the critics have for this, or which actuaries were not aware that the stock market rise of the late 1990s was an unsustainable bubble. Certainly many were so aware.

Calculation 2

In paragraph 11 of the Executive Summary you say: "Against this, actuarial expertise must not be confused with an ability to forecast the future. Moreover, an actuary who, in the early 1990s, persisted with forecasts of inflation and interest rates that in the event turned out to be correct would at that time have lost a substantial amount of credibility. But actuarial work emphasising such outcomes as (rather remote) possibilities would have been unlikely to have much impact."

It is interesting to consider the public attitude to risks whose probability may be low, but far from negligible. As a different actuarial example I consider the probability that the present Queen (a female born in 1926), might survive her son Charles (a male born in 1948). This is a straightforward traditional actuarial calculation that depends on the mortality tables assumed for the two individuals. I assume that both are healthy; there is no public evidence to the contrary. I use the mortality tables that I would consider most appropriate, those called "RFC92" and "RMC92", both based on the experience for the years 1991-1994 of those who had purchased self-employed retirement annuities from life offices (who are therefore in the upper levels of income), taking both deferred and vested together (the "combined experience"). I use the forecast made in the 1990s (but not the more recent ones issued by the CMI Bureau), and allow for the mortality for the two individual's years of birth (i.e. using RFC92B1926 and RMC92B1948).

I find on this basis that the probability that Prince Charles predeceases his mother is about 10%. If I use the latest published English Life Tables, ELT15M and ELT15F, based on the population for England & Wales for 1990-92 the probability changes to about 16%. Of course it is more likely that she will die first, but I have seen no comment in the newspapers about the possibility of the opposite happening, or the consequences of this. Do journalists not consider these calculations, or are they not interested in possibilities of lowish, but not negligible probability? They are interested enough sometimes in possible events with extremely low probabilities, but serious or dramatic outcomes.

I hope that these remarks are of some interest and use to you. I have no objections to their being publicised. I am very happy to elaborate on them in any way you might wish.

Yours sincerely

A D Wilkie