A theoretical note on cross-border interactions between carbon abatement schemes

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1. Introduction

1.1 As countries address the issue of carbon abatement in different ways and at different speeds it is important to understand how interactions between policy regimes will impact on such schemes. The purpose of this analysis is to provide some basic guidance on how interactions between countries or sectors will affect the resulting output of emissions, distribution of abatement activity and government revenues.

1.2 The starting point for this paper is a benchmark model of two countries with separate policies for emissions abatement. This benchmark model and the notational conventions adopted are introduced below, followed by some discussion of the outcomes - such as emissions levels - that are of interest. Subsequent sections examine variants of this model with different assumptions about policy choices and underlying parameters.

1.3 This paper examines what happens when countries adopt different taxes, subsidies, permit trading schemes or regulatory instruments. It is important to understand interactions both between similar instruments applied more or less tightly (e.g. different tax rates) and also between different instruments altogether. For example how does the presence of a low carbon tax in one country impact on a neighbour's permit scheme?

1.4 In addition to the usual concerns about determining optimal emissions levels and distributing abatement activity, a multi-track approach can raise problems for the effectiveness of individual policies as firms relocate to different jurisdictions or consumers substitute between goods. For example a tax rise in one country will be far less effective at reducing overall emissions if it is easy for firms to escape the tax by relocating production.

The Benchmark Model

1.5 Two countries A and B face domestic demand for emissions rights $F_a(?)$ and $F_b(?)$ and inverse demand $P_a(?)$ and $P_b(?)$. Producers in the two countries are not competitors in the product or capital markets and only interact in the market for emissions rights if at all. Within each country producers are perfectly competitive so that domestic welfare depends only on consumer surplus and government revenues.

1.6 In the benchmark model the marginal social costs of emissions C_a and C_b are the same across countries. The costs of emissions that are confined to a single country are also assumed to be insignificant in the absence of policy, so that the model does not address *ancillary benefits* of abatement such as reduced local air pollution.[1] There is no uncertainty about the social costs of emissions or the costs of abatement.

1.7 Trade of emissions permits may be possible between countries. In the benchmark model there is no cross border trade but firms may trade permits within countries. If

[1] This may be a reasonable approximation for carbon dioxide but ancillary benefits may be larger for other emissions. The implications of differing social costs of emissions C_a ? C_b are considered in Part III.

countries impose taxes then these are imposed on emissions from the domestic production of domestically consumed goods.

1.8 In the benchmark model all domestically consumed goods are produced locally and there is no opportunity for cross-border trade in goods or factor relocation. However in other versions of the model firms with low relocation costs may choose to shift production in order to exploit differences in policy.[2] Factor and product mobility will depend on transportation costs and the ease of relocation.

1.9 Border adjustments on permits or the trade of goods could be used to mitigate cross border differences in emissions policy. These are redundant in the benchmark model but will be considered in later versions of the model.

1.10 Box 1 summarizes the outcomes in the benchmark model with permits or taxes on domestic emissions. In this benchmark model the same outcomes can be achieved with either the price instrument (taxes) or the quantity instrument (permits).

Box 1. Benchmark model

With permits

$C_a = C_b = C$
Q_a permits in A
Q_b permits in B
No trade
No factor/product mobility
No border adjustments

Emissions
A: Q_a
B: Q_b
Total: $Q = Q_a + Q_b$

Revenues
A: $Q_a P_a(Q_a)$
B: $Q_b P_b(Q_b)$
Total: $Q_a P_a(Q_a) + Q_b P_b(Q_b)$

Production
Conditional on Q efficiency of abatement requires $P_a = P_b$
First best requires $P_a = P_b = C$

With taxes

$C_a = C_b = C$
Tax T_a in A
Tax T_b in B
No trade
No factor/product mobility
No border adjustments

Emissions
A: $F_a(T_a)$
B: $F_b(T_b)$
Total: $F_a(T_a) + F_b(T_b)$

Revenues
A: $T_a F_a(T_a)$
B: $T_b F_b(T_b)$
Total: $T_a F_a(T_a) + T_b F_b(T_b)$

Production
Conditional on emissions efficiency of abatement requires $T_a = T_b$
First best requires $T_a = T_b = C$

Outcomes of the Model

[2] When producers shift to countries with relatively lax abatement policies and thereby mitigate the effect of a strict abatement policy this is sometimes known as carbon leakage. For example if a policy leads to a decrease of 10 tonnes of carbon emissions in country A but an increase of 2 tonnes in country B the leakage rate is 20%.

Emissions

1.11 This paper examines the effect of policy interactions on the aggregate level of emissions as well as the domestic level of emissions in each country. In the benchmark model the first best level of emissions is found where the marginal social costs and benefits of emissions are equal. When $C_a = C_b = C$, and assuming that there are no ancillary private benefits of abatement, this implies that the price of increasing emissions in each country is equal to C.

Revenues

1.12 Interactions between different policy regimes are likely to affect the distribution and level of government revenue (or expenditure). Only emissions-related revenue effects are included here although other revenues and expenditures such as corporation taxes could also be affected by abatement schemes and ensuing interactions between policy regimes.

1.13 The revenue calculations in this paper assume that permits are sold to producers and therefore represent an upper limit to the revenues that the governments could generate from a permit scheme. If a government chooses to forgo such revenues (i.e. by grandfathering permits) actual revenues may be lower.

Efficiency of Abatement

1.14 In reducing the level of global emissions it is desirable to minimize the total social cost of abatement. Productive efficiency requires that in achieving any given level of emissions Q the marginal cost of abatement is equal across countries.[3] Otherwise abatement could be transferred between countries, lowering costs without affecting total emissions. Thus conditional on any given level of overall emissions the efficient distribution of abatement requires that *the price of emissions rights is equalized across countries*.[4] This principle is derived formally in the appendix.

Output, Consumer Surplus and Producer Rents

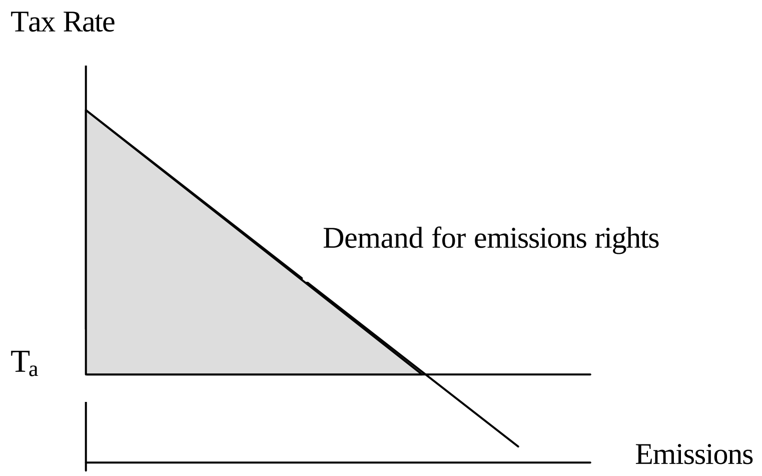
1.15 When the relationship between emissions and production is constant the distribution of emissions will also describe the relative allocations of carbon-emitting output between countries. For example if one unit of output in country A produces R_a units of emissions then output Y_a can be inferred from the level of emissions (i.e. $Y_a = F_a/R_a$). However this relationship need not be constant if abatement activities affect the technology of production (e.g. R_a changes). Caution must therefore be taken in interpreting the emissions level as an indicator of effects on output.

1.16 In addition to the level of emissions and the allocation of production governments may be concerned with the distribution of consumer surplus and producer rents. Assuming that the market for emissions is competitive the joint consumer and producer surplus is given by the difference between what producers would be willing to pay and what they actually pay for each unit of emissions. In the case of a tax in country A this surplus is represented graphically by the shaded area under the demand curve in Figure 1 below.

[3] This assumes that the costs of emissions are equal across countries. This is probably a plausible assumption in the case of carbon dioxide, but the implications of relaxing this assumption are explored in Part III.

[4] Concerns about distributive efficiency are suppressed in this paper in order to maintain clarity of the exposition.

Figure 1: Surplus



1.17 How this surplus is allocated between consumers and producers will depend on market structure. If production is perfectly competitive as assumed in the benchmark model then no rents accrue to firms. On the other hand market power may allow firms to appropriate some of this surplus.[5] In so far as carbon-emitting production yields consumer or producer surplus, governments are likely to face pressure to limit abatement.

1.18 From Figure 1 it is clear that an increase in the price of emissions (through permit price rises or tax increases) will reduce joint surplus and the level of emissions used in supplying goods for country A. However care must be taken in inferring effects on surplus from emissions levels. If there is trade in goods the level of emissions geographically situated in country A could include production of goods for consumers in B and vice versa.[6]

Plan of Paper

1.19 Part I discusses the major instruments of carbon abatement: permits, taxes, subsidies, and regulation. Part II looks at three other policy choices: price ceilings or floors, simultaneous taxes and permits, and border adjustments. Part III reconsiders the assumptions of the benchmark model including: the cost of factor mobility, competition in the product market, and asymmetric costs of emissions. The appendix summarizes in a more formal manner some of the results discussed in the text.

[5] Note that in this case the demand curve for emissions may shift as producers limit production or raise the prices of final goods.
[6] Producer surplus will also be affected if there is competition between producers of goods for each country (see section 10).

Part I Basic Principles of Regime Interaction

2. Permits and Permit Trade

2.1 The principle of equal emissions prices described above implies that two entirely separate permit schemes will result in inefficient abatement. Except in the special case that initial permit levels satisfy $P_a(Q_a) = P_b(Q_b)$ prices will not be the same and producers will face different marginal benefits from abatement.

2.2 Permit trade between countries resolves this problem and leads to improvements in the efficiency of abatement. When permits are fully tradable the single permit price $P(Q)$ depends on the total level of emissions $Q = Q_a + Q_b$. Conditional on the level of emissions and the corresponding permit price production is now allocated efficiently between the two countries. The first best level of emissions Q^* satisfies $P(Q^*) = C$.

Box 2. Tradable permits

$C_a = C_b = C$
Q_a permits in A
Q_b permits in B
Tradable permits
No factor/product mobility
No border adjustments

Emissions/Output
A: $F_a(P(Q))$
B: $F_b(P(Q))$
Total: Q

Revenues
A: $Q_aP(Q)$
B: $Q_bP(Q)$
Total: $Q_aP(Q) + Q_bP(Q)$

Production
Conditional on Q the price P(Q) is efficient
First best requires $Q^* = P^{-1}(C)$

2.3 Since the quantity of permits sold by each country is fixed the change in revenues when trade is introduced depends only on the change in local permit prices.[7] One country will experience an increase in prices and revenues while the other will experience a fall in prices and revenues. Emissions will increase in the country whose prices fall as producers consume permits bought abroad, while emissions fall in the country whose prices rise. Production may be affected in the same way but this will depend on whether abatement involves a reduction in production or simply an improvement in efficiency.

[7] Of course if countries decide to coordinate in setting permit quantities revenues will depend on the quantities that are negotiated.

2.4 Firm relocation can achieve similar results to allowing cross border trade in permits. If permits are not tradable and there is costless factor mobility and free trade of final goods then the permit prices in A and B are equalized through firm relocation. Prices, revenues and efficiency are the same as in the case of tradable permits. The only difference is that emissions are now confined to the country issuing the permit used.

2.5 If there are positive costs of relocation there may remain some difference in permit prices and therefore some productive inefficiency. Moreover the costs of relocation represent a deadweight loss that could be avoided in the case of tradable permits.

2.6 With both tradable permits and free relocation there is a continuum of equilibrium outcomes all at the single permit price P(Q) but with different combinations of relocation and permit trade. However it is likely that the costs of relocation are strictly greater than those of permit trade so that the most probable outcome has trade but no relocation.

3. Taxes and the Tax Base

3.1 The principle of equal emissions prices also implies that two entirely separate tax schemes will result in an inefficient distribution of abatement unless the tax rates are identical $T_a = T_b$. Furthermore the first best level of emissions also requires that the marginal costs of abatement (the tax rates in equilibrium) are equal to the marginal social benefit of abatement C. The remainder of this section considers the case of $T_a < T_b$.

3.2 The interaction between two tax regimes depends critically on the tax base employed in each scheme and how each country treats producers caught in multiple tax bases. A country's tax base could be defined in a number of ways including:

1) emissions from the production of domestically consumed goods
2) domestic emissions irrespective of the final destination of goods[8]
3) emissions of domestically incorporated companies wherever they produce or sell

3.3 As long as the countries employ the same definition of the tax base from the list above then there is no overlap in the tax base. However there are differences in the incentives for relocation. In the first case taxation of emissions from domestically consumed goods prevents firms from benefiting from relocation. In the second and third cases there may still be an incentive for relocation if the tax rates differ.

3.4 This is likely to occur if producers are able to change the tax base they belong to by relocating factors of production and moving goods across borders. If it is costless to move production to another country and there continues to be no border adjustment for imported goods then producers will relocate to the lowest tax jurisdiction, country A. Total emissions rise since the tax rate on B's producers falls from T_b to T_a. Tax revenues rise in A and fall to zero in B and all carbon emissions take place in A.[9] Furthermore because of relocation it is not necessarily true that a rise in emissions prices (i.e. the tax rate) in one country will lead to a fall in total emissions levels.

3.5 If firms fall into overlapping tax bases their treatment can also vary considerably. For example the countries' governments could require that:

[8] Monitoring requirements may well make this the most straightforward choice of the tax base.
[9] For higher costs of relocation some production may remain in B despite higher taxes.

1) the producer chooses which tax to pay
2) the producer is taxed twice
3) the producer is required to pay the higher of the two taxes, with revenues distributed between governments according to an agreed formula [10]

3.6 Producer choice will allow firms in both tax bases to pay the lower tax rate T_a. Revenues and total emissions will be similar to the case of relocation, but firms will not need to relocate to take advantage of more lenient policy. If both tax regime choice and costless factor mobility are possible all firms will once again pay T_a and any allocation of emissions is possible.[11] However it is likely that relocation costs are positive in which case firms will strictly prefer not to relocate.

4. Subsidies

Direct Subsidies

4.1 Direct subsidies for reducing carbon emissions work in a similar way to taxes in achieving emissions targets and inducing abatement. In deciding whether to reduce emissions producers are concerned with the difference in the prices of emitting and not emitting, and it doesn't matter to production decisions whether this is achieved through a tax or a subsidy.

4.2 However the distribution of revenues from a subsidy is very different from that of a tax. In a system of subsidies the government's revenue is negative and producers receive the entire surplus. Therefore if country A operates a tax on emissions and B employs an equal subsidy on emissions reductions the producers will not be indifferent between the two schemes even though the marginal costs of emitting in each country are the same.

4.3 If producers fall into both tax/subsidy bases and can choose which regime to participate in they will clearly prefer to receive subsidies for abatement. Similarly if relocation is costless producers will shift the production of emissions to the subsidising regime. Subsidies provide an even greater incentive for relocation than lower taxes. Revenues in A will fall to 0 and B will spend even more on subsidising abatement.[12] Overall emissions should not change from the benchmark case.

4.4 Although direct subsidies can attain efficient outcomes in principle there are likely to be considerable practical difficulties in implementing such a policy. For example firm-level data on emissions may be unavailable making it difficult for the subsidising authority to establish the benchmark against which abatement is measured. Moreover there may be an incentive for firms to expand present output and emissions if they anticipate that direct subsidies will be introduced based on reductions from current levels. In some cases firms may even enter the market in order to receive subsidies. Such incentives are likely to result in inefficient production decisions, and will mitigate the effects of emissions policy.

[10] This is equivalent to a scheme with deductions for foreign taxes paid.
[11] When factor mobility is costless the producers are all indifferent between locations.
[12] Of course corporation taxes from relocating producers may offset the extra subsidies paid.

Box 3. Direct Subsidies and Taxes	
Without relocation	***With relocation***
$C_a = C_b = C$	$C_a = C_b = C$
Tax T in A	Tax T in A
Subsidy T for abatement in B	Subsidy T for abatement in B
No trade	No trade
No factor/product mobility	Costless factor/product mobility
No border adjustments	No border adjustments
Emissions	*Emissions*
A: $F_a(T)$	A: 0
B: $F_b(T)$	B: $F_a(T) + F_b(T)$
Total: $F_a(T) + F_b(T)$	Total: $F_a(T) + F_b(T)$
Revenues	*Revenues*
A: $TF_a(T)$	A: 0
B: $- TF_b(T)$	B: $- TF_a(T) - TF_b(T)$
Total: $TF_a(T) - TF_b(T)$	Total: $- TF_a(T) - TF_b(T)$
Production	*Production*
First best emissions implies T = C	First best emissions implies T = C

Indirect Subsidies

4.5 As it can be difficult to subsidise emissions reductions directly governments often employ indirect subsidies for technologies of production with low emissions or for technologies that reduce the emissions from existing methods of production.

4.6 First the subsidy could make it more attractive to use a different production technology with lower emissions per unit output (e.g. a subsidy for the use of renewable energy sources). If there are only two possible production technologies each with a different constant level of emissions per unit of production the total level of emissions could be expressed as:

$$F_a(T_a , k_{a,2}) = R_{a,1} Y_{a,1}(T_a , k_{a,2}) + R_{a,2} Y_{a,2}(T_a , k_{a,2})$$

Where $R_{a,1} > R_{a,2}$ are the respective levels of emissions per unit output and $k_{a,2}$ denotes the cost of the cleaner technology.[13] Lowering $k_{a,2}$ through a subsidy lowers $Y_{a,1}$ and raises $Y_{a,2}$. Overall emissions will fall as long as $e_{12} > (R_{a,2}\ Y_{a,2}) / (R_{a,1}\ Y_{a,1})$ where $e_{12} = (\partial Y_{a,1}/\partial Y_{a,2})\ (Y_{a,2}/ Y_{a,1})$ denotes the elasticity of substitution between the two sources of production.

4.7 What this means is that for the indirect subsidy to be effective at reducing emissions, producer behaviour has to be sensitive to the change in the cost of the more efficient

[13] The cost of the less clean technology is suppressed for notational clarity.

technology. Reducing the cost of the cheaper technology encourages two things: first there is increased production due to the lowering of overall costs and second there is switching of production between technologies because of changing relative costs. Emissions fall as long as the switching effect dominates and this relies on producers' choices between technologies being sensitive to the relative costs of production. The greater the proportion of emissions already accounted for by the efficient technology (i.e. when the ratio $(R_{a,2}\ Y_{a,2}) / (R_{a,1}\ Y_{a,1})$ is higher) the greater must be the propensity for the remaining inefficient producers to switch.

4.8 Second the subsidy could reduce the cost of cleaning up emissions from existing production technologies (e.g. carbon capture and storage). In this case the level of emissions F_a depends on output Y_a and the cleaning technology R_a. Each of these functions depends in turn on the tax level and the cost of the cleaning technology K_a. For example:

$$F_a(T_a , K_a) = R_a(T_a , K_a)\, Y_a(T_a , R_a(T_a , K_a))$$

4.9 Unlike subsidies for alternative production technologies these subsidies for cleaning technologies require an incentive such as a tax for any cleaning to be worthwhile for firms. Lowering K_a leads to a decrease in the emissions per unit output $(\partial R_a/ \partial K_a) > 0$ and a corresponding increase in the level of output because $(\partial Y_a/ \partial R_a) < 0$. Overall emissions fall as long as $e_{YR} < 1$ where $e_{YR} = -(R_a/ Y_a)(\partial Y_a/ \partial R_a)$ denotes the elasticity of demand with respect to the efficiency of output.[14] In this case the subsidy leads to lower overall emissions as long as the lower costs of production (i.e. lower taxes paid per unit output) do not induce a large increase in demand. For example a subsidy for more fuel efficient engines could reduce the running costs of a car and cause an increase in car ownership. If this increase is large enough the overall effect on emissions could be positive.

4.10 In both cases the use of indirect subsidies leads to inefficient production. Even though the first best level of emissions may be achieved through the use of such subsidies, the level of production (as opposed to emissions) is likely to be inefficiently high. Subsidies can achieve the correct *relative* pricing of different production technologies such as coal-fired power stations and wind farms, but the *absolute* level of prices for both technologies is inefficiently low.[15]

4.11 Now suppose that B operates a tax on emissions $T_b > T_a \geq 0$. Costless factor relocation would allow all carbon-emitting production to shift from B to A. Revenues in B would correspondingly fall to zero and revenues in A would also fall if subsidies to relocating firms outweigh any tax revenues from them.[16] If emissions are at the first best level overall production is likely to be even higher than in the absence of relocation (with cleaning subsidies) or even more inefficiently weighted towards cleaner production technologies. Trade in emissions rights (or tax and subsidy

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[14] In the same model an increase in taxes will lead to a fall in emissions as long as e_{YR} satisfies a less stringent condition, so there may be situations in which a marginal increase in the tax is effective at reducing emissions and a marginal increase in the subsidy is not.

[15] For example the first-best outcome may require both that some production is switched to a more clean technology and also that total production is reduced. Subsidies for cleaner technologies encourage switching by changing the relative costs of technologies but discourage aggregate reductions because the costs of production have fallen.

[16] Note that this analysis is confined to emissions-related taxes and revenues. It is possible that emissions-related revenues are negative while overall tax revenues from relocating firms are positive (e.g. because of profits tax).

regime choice) has similar effects except that production does not need to relocate to take advantage of subsidies or lower taxes.

Research Subsidies

4.12 To the extent that learning-by-doing is a substantial element of research, the indirect abatement subsidies discussed above may also be treated as research subsidies. In this case short-term inefficiencies in production may be outweighed by long-term gains in efficiency.

4.13 Subsidies for research and development are a largely separate policy concern in that these subsidies only affect the activities of a relatively small number of firms in the short term. However subsidies for adoption of innovative technologies should be considered in the same way as the indirect subsidies above.

5. Regulation of Emissions

5.1 Regulation of emissions is common and can take a variety of forms. In a command-and-control scheme the government could stipulate the exact level of emissions from each producer. This is equivalent to a permit scheme without trade even between domestic firms. In this case the implied shadow price of emissions rights will be equal to the marginal cost of abating to the regulated level and will differ between firms as well as between countries.

5.2 Less directly the government could impose a minimum efficiency standard for production or ban the use of certain technologies altogether. For example consider the two technology model of section 4:

$$F_a(T_a , k_{a,2}) = R_{a,1} Y_{a,1}(T_a , k_{a,2}) + R_{a,2} Y_{a,2}(T_a , k_{a,2})$$

Regulations could require $Y_{a,1}(T_a , k_{a,2}) = 0$ which is equivalent to imposing an infinitely high tax on $Y_{a,1}$. Alternatively in the variable efficiency model of section 4 the government could impose a minimum level for $R_a(T_a , K_a)$ in the emissions function:

$$F_a(T_a , K_a) = R_a(T_a , K_a)\, Y_a(T_a , R_a(T_a , K_a))$$

5.3 Again the shadow price of emissions is likely to vary considerably even across firms. The costs of shifting between production technologies or improving the efficiency of existing technologies will vary and so regulation will impose different costs of abatement on firms.[17]

5.4 If relocation is costless firms can escape regulation in country A by moving some production to country B.[18] Producers will only relocate production if the shadow price of emissions in A is higher than the actual price (permits or taxes) payable in B. Moreover the firms will relocate a proportion of production up until the point that shadow prices and prices are equalized. Some firms would continue to operate with a lower shadow price of emissions in A and would not relocate any production. Revenues would increase in B as the permit price rose due to increased demand or as the number of firms paying taxes increases.

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[17] Unlike indirect subsidies, regulation imposes the costs of compliance on the firms and so revenues are not affected.

[18] For the time being it is assumed that firms in B cannot relocate because they would not be allowed any emissions under A's regulatory regime.

5.5 In spite of these productive inefficiencies informational constraints may make regulation desirable for certain products or industries. In this case care should also be taken to ensure that goods imported by competitors or relocated firms are consistent with the objectives of the regulation.

Part II Other Policy Instruments

6. Permit Price Ceilings and Floors

6.1 In a single-country permit scheme price floors and ceilings are enforced by reducing or expanding the domestic supply of permits. In the absence of trade or factor mobility different countries can choose price floors and ceilings independently according to domestic requirements. The efficiency of the distribution of abatement will depend on the permit prices in each country as in the case of permits without price ceilings or floors.

6.2 When two permit schemes merge and allow cross-border trade a single market price for permits will emerge. If governments wish to enforce a price floor or ceiling through the supply of permits they will have to influence the overall market price thereby imposing the same floor or ceiling on the other country.

6.3 If both countries enforce price ceilings and floors the tightest bounds will affect the price and affect the total quantity of emissions. Denoting the price ceilings of A and B respectively as $P_{a,h}$ and $P_{b,h}$ and the price floors as $P_{a,l}$ and $P_{b,l}$ the permit price when there is trade can be expressed more formally as:

$$\text{Price} = \begin{cases} P(Q) & \text{if } \min\{P_{a,h}, P_{b,h}\} \geq P(Q) \geq \max\{P_{a,l}, P_{b,l}\} \\ \max\{P_{a,l}, P_{b,l}\} & \text{if } P(Q) < \max\{P_{a,l}, P_{b,l}\} \\ \min\{P_{a,h}, P_{b,h}\} & \text{if } \min\{P_{a,h}, P_{b,h}\} < P(Q) \end{cases}$$

6.4 Price equalization implies that conditional on the level of emissions the distribution of abatement will be efficient. Revenues may vary considerably and could even be negative for one of the countries in spite of the positive price of permits. For example one country may sustain a price floor by buying permits issued by the other country while issuing no permits of its own.

7. Taxes and Permits

7.1 The principle of price equalization continues to apply when countries adopt different policy instruments. Suppose that country A imposes a tax T_a and country B issues permits Q_b. In the benchmark model without trade or firm relocation the overall emissions level depends on Q_b and $F_a(T_a)$. The prices of emitting in the two countries are likely to differ unless Q_b is chosen such that $P_b(Q_b) = T_a$. Unequal prices lead to an inefficient allocation of production and the first best outcome requires $P_b(Q_b) = T_a = C$.

7.2 Suppose that permits are freely traded but that only producers in country A can choose to pay the tax instead. There are three cases to consider depending on the relative strength of demand for emissions rights in the two countries. If the price of permits in B is already higher than the tax rate $P_b(Q_b) > T_a$ then there is no permit trade and firms in A opt to pay the tax. If the price of permits is lower than the tax rate even when all the original tax paying firms buy permits $P_b(Q_b + F_a(T_a)) < T_a$ then no producers pay the tax and the price P lies in the range $P_b(Q_b + F_a(T_a)) < P < T_a$. In the intermediate case the permit price and tax rate are equalized at T_a and abatement is efficiently distributed conditional on the overall level of emissions.

7.3 If permits are tradable and all firms can also choose to pay the tax instead the two prices are equalized at T_a unless $P_b(Q_b + F_a(T_a)) < T_a$ in which case no taxes are paid and the price P lies in the range $P_b(Q_b + F_a(T_a)) < P < T_a$. This is equivalent to a permit scheme with an initial allocation of Q_b permits supplied by B and a price ceiling of T_a enforced by A. The tax sets the price of emissions rights while the quantity of permits decides the allocation of revenues between A and B. Similar results arise if abatement schemes are confined to domestic production but firms can relocate between countries, except that the distribution of emissions will be different.

Simultaneous Taxes and Permits

7.4 It would of course be possible to impose both taxes and permits on a country or sector simultaneously. The government could issue a capped quantity of permits and also tax any producer who consumed a permit. In the benchmark model this is similar to setting a price floor on the overall cost of emissions (although permit prices themselves are unprotected). If A imposes a tax T_a as well as Q_a permits the permit price $P_a(Q_a , T_a)$ is equal to $\max\{P_a(Q_a , 0) - T_a , 0\}$ where $P_a(Q_a , 0)$ is the price of permits in the absence of the extra tax. The overall price of emissions in A is the same as in the benchmark model except with the addition of a net price floor of T_a.

7.5 If country A is joined in a permit trading scheme by B the tax T_a is no longer equivalent to the price floor discussed in section 6. The producers in A are still subject to a price floor of T_a but the producers in B are not. In fact the producers in B will face a lower price for permits than when there is a price floor of T_a (achieved by A reducing the number of permits in the market). With trade the price for permits is equalized but the net prices paid by producers in A and B are different. Given equal social costs of emissions this leads to an inefficient distribution of abatement.

7.6 The effects of costless factor mobility on permit prices differ again from those of tradability. In this case net emissions prices (including taxes) are equalized across countries unless there are no emissions in A at all (i.e. $P(Q_b) < T_a$). The effective price floor of T_a applies now to A's permits rather than to A's producers.

7.7 If there is both trade in permits and costless relocation then all emissions will relocate to country B. This is because price equalization for permits ensures that the total price for emissions rights is always higher in A.

Box 4. Simultaneous Permits and Taxes

With permit trade	***With factor mobility***
$C_a = C_b = C$	$C_a = C_b = C$
Q_a permits issued by A	Q_a permits issued by A
Q_b permits issued by B	Q_b permits issued by B
Tax T_a also imposed by A	Tax T_a also imposed by A
$P(Q , T_a)$ is price of permits given T_a	$P(Q , T_a)$ is price of permits given T_a
Tradable permits	No Trade
No factor/product mobility	Costless factor/product mobility

No border adjustments	No border adjustments
Emissions	*Emissions*
A: $F_a(P(Q, T_a) + T_a)$	A: Q_a if $P_a(Q_a, T_a) > 0$ $F_a(T_a)$ if $P_a(Q_a, T_a) = 0$ 0 if $P(Q_b) < T_a$
B: $F_b(P(Q, T_a)$	B: Q_b
Total: $Q = Q_a + Q_b$	Total: $Q = Q_a + Q_b$
Revenues	*Revenues*
A: $T_a F_a(P(Q, T_a) + T_a) + Q_a P(Q, T_a)$	A: $Q_a P_a(Q_a, T_a) + Q_a T_a$ if $P_a(Q_a, T_a) > 0$ $T_a F_a(T_a)$ if $P_a(Q_a, T_a) = 0$ 0 if $P(Q_b) < T_a$
B: $Q_b P(Q, T_a)$	B: $Q_b P_b(Q_b)$
Production First best requires $P_a = P_b = C$ However $P_a = P(Q, T_a) + T_a$ and $P_b = P(Q, T_a)$ so inefficient	*Production* Price equalization $P_a(Q_a, T_a) + T_a = P_b(Q_b)$ (or no taxes paid at all)

8. Border Adjustments

8.1 Border adjustments can be used to mitigate policy asymmetries, and to reduce incentives for domestic production to be relocated abroad or replaced by foreign competition.[19]

8.2 Reconsider the case in which both countries impose emissions taxes and that $T_b > T_a$. As shown in section 3 this creates incentives for producers to relocate to country A, thereby reducing B's revenues and leading to a higher level of emissions. Now suppose that B imposes an import tax of $\Delta \leq T_b - T_a$ on all imports subject to the lower tax rate T_a. Then if the strict inequality $\Delta < T_b - T_a$ holds firms (with zero relocation costs) will still relocate from B to A to take advantage of lower taxes.[20] However revenues for B will not fall to 0 as the government raises revenue of $\Delta F_b(T_a + \Delta)$ from the border adjustment, and the level of emissions from producers serving B's market will only increase to $F_b(T_a + \Delta)$ which is less than the corresponding emissions in the absence of the adjustment $F_b(T_a)$.

8.3 On the other hand A could choose to impose an export tax on goods exported to B. The outcome would be equivalent to the import tax case described above except that A retains the revenues from the border adjustment. For the same size of tax (i.e. Δ) the emissions level is unaffected.

Box 5. Taxes, Factor Relocation and Border Adjustments

Import Tax in B	***Export Tax in A***

[19] As shown in section 10 product market competition can be analyzed in much the same way as factor relocation.

[20] If $\Delta = T_b - T_a$ exactly then producers are indifferent between relocating and paying import duties and not relocating at all. The equilibrium emissions level in B could then lie anywhere between 0 and $F_b(T_b)$.

$C_a = C_b = C$	$C_a = C_b = C$
$T_a < T_b$	$T_a < T_b$
No trade in emissions rights	No trade in emissions rights
Costless factor/product mobility	Costless factor/product mobility
Import tax $\Delta < T_b - T_a$ imposed by B	Export tax $\Delta < T_b - T_a$ imposed by A
Emissions	*Emissions*
A: $F_a(T_a) + F_b(T_a + \Delta)$	A: $F_a(T_a) + F_b(T_a + \Delta)$
B: 0	B: 0
Total: $F_a(T_a) + F_b(T_a + \Delta)$	Total: $F_a(T_a) + F_b(T_a + \Delta)$
Revenues	*Revenues*
A: $T_aF_a(T_a) + T_aF_b(T_a + \Delta)$	A: $T_aF_a(T_a) + (T_a + \Delta)F_b(T_a + \Delta)$
B: $\Delta F_b(T_a + \Delta)$	B: 0
Total: $T_aF_a(T_a) + (T_a + \Delta)F_b(T_a + \Delta)$	Total: $T_aF_a(T_a) + (T_a + \Delta)F_b(T_a + \Delta)$
Production	*Production*
First best requires $T_a = C$ and $\Delta = 0$	First best requires $T_a = C$ and $\Delta = 0$

8.4 Border adjustments can also take the form of subsidies. If country B wishes to increase domestic production it could make relocation attractive to A's producers by offering an export subsidy of $\Delta > T_b - T_a$, in effect taxing A's producers at a rate below T_a. Similar subsidies could be used if producers compete across borders.

8.5 Although border adjustments can be used to mitigate tax asymmetries they can also be used to exacerbate these asymmetries. For example import duties could be imposed on goods already subject to higher emissions taxes or goods exported from lower tax regimes could be granted an export subsidy.[21]

8.6 Border adjustments may sometimes be difficult to reconcile with direct regulation or voluntary restrictions on emissions. For example export subsidies that encourage increased output may undermine any voluntary restrictions that are in place. Moreover it may be difficult to decide on the appropriate level for duties when the shadow price of emissions rights varies across firms.

Part III Further Observations

9. Factor Mobility and Trade

9.1 Factor relocation (and subsequent trade in goods) can act as a force for convergence in emissions prices. In the permit schemes discussed in section 2 costless factor mobility leads to permit price equalization. Similarly with taxes in section 3 costless relocation allows all producers to pay the lower tax rate, hence the actual price paid for emissions is the same for all producers. The overall desirability of this convergence will depend on the relative importance of two effects: the distribution of abatement activity and the change in overall abatement levels.

[21] Trade policy involves a far greater range of considerations than emissions policy alone. While this section provides some basic guidance on the interaction of border adjustments with emissions policy, it does not purport to comment on the desirability or otherwise of border adjustments in general or in any specific instance.

9.2 However if relocation and trade is not costless or if some producers are unable to relocate, it is not necessarily true that emissions prices will converge.[22] In practice it is likely that the costs of relocating production will be positive and also vary across firms, and so this section examines the impacts of intermediate and heterogeneous factor mobility.

9.3 If firms face positive but finite costs of relocation then relocating production in order to exploit policy differences induces a deadweight loss in social welfare. It could be more efficient to allow those firms contemplating relocation to participate in other countries' schemes for example through permit trade.

9.4 If some goods are fully tradable (with costless relocation) and other goods are not tradable at all then the ability of relocation to achieve price equalization will be moderated. Below are shown the results for partial relocation in the benchmark model with taxes.

Box 6. Taxes with Partial Relocation

$C_a = C_b = C$
$T_a < T_b$
No trade
Costless factor/product mobility with probability $f = 1$ for any firm in B
No border adjustments

Emissions/Output
A: $F_a(T_a) + f\,F_b(T_a)$
B: $(1 - f)F_b(T_a)$
Total: $F_a(T_a) + F_b(T_a)$

Revenues
A: $T_aF_a(T_a) + f\,T_aF_b(T_a)$
B: $(1 - f)T_aF_b(T_a)$
Total: $T_aF_a(T_a) + T_aF_b(T_a)$

Production
Conditional on emissions level efficiency of abatement requires $T_a = T_b$ unless $f = 1$
First best requires $T_a = T_b = C$ if $f < 1$or $T_a = C$ if $f = 1$

9.5 Empirically f is likely to be relatively low given that non-tradables such as energy and transport comprise a large proportion of carbon intensive industries. The scope for emissions price equalization through relocation will then be limited.

10. Product Market Interactions and Competitiveness

10.1 The benchmark model assumes that producers in countries A and B do not compete in the product market and only interact if at all in the market for emissions rights.

[22] For example if all production is initially located in B and then half the producers relocate to A and pay lower taxes, the emissions prices paid by firms clearly diverge.

However competitiveness is a major concern of governments and is likely to affect the preferred choice and extent of abatement policies.

10.2 Product market competition works in a similar way to factor relocation discussed in preceding sections. Rather than production being relocated and goods re-imported by domestic producers, competition allows this role to be performed by overseas producers. Production is relocated between countries via a change in producer rather than the physical relocation of an existing supplier.

10.3 If goods are perfect substitutes, and assuming identical and constant (non-tax) marginal costs of production and costless trade, the impact of competition will be very similar to that of costless factor mobility. Output of emissions will shift to producers in countries with lower prices for emissions until prices are equalized (in the case of permits) or all emissions are located in one country (in the case of taxes). Like factor relocation competition can act as a force for convergence in the prices paid for emissions.[23]

10.4 One important difference between factor relocation and product market competition is in the distribution of producer rents. If markets are imperfectly competitive and producers earn positive rents then costless factor relocation (without competition between producers in different countries) will only affect the size of the rents accruing to producers by changing output at the margin. On the other hand competition between producers could also result in inframarginal rents being transferred to producers in other countries.

11. Asymmetric Costs of Emissions

11.1 A general theme of this analysis has been that price equalization leads to efficient allocations of production conditional on the overall level of emissions chosen. However this conclusion relies on the assumption that $C_a = C_b$. If externality costs are not equal $C_a \neq C_b$ then unimpeded trade (of either permits or taxes) or costless factor mobility is likely to lead to inefficient production and emissions decisions. Trade and mobility ensure that a single permit price or tax is paid by producers. If $C_a \neq C_b$ then the single price cannot internalize the costs of emissions.

11.2 Such asymmetries may arise because taxes and permits must be applied to intermediate tax bases such as fossil fuel sales rather than to the ultimate environmental harm caused by emissions. For example if the manner or location of fuel consumption affects the impact or level of emissions from those fuels, then a single permit price or tax may be inefficient. Appropriate weightings could be introduced between sectors (like a border tax adjustment between countries) but may be difficult to implement and enforce.

12. Conclusion

Because of the large number of possible combinations of different assumptions this paper cannot be an exhaustive account of policy interactions. However a few general principles can be extracted from the preceding analysis.

General Principles

[23] And once again relaxing an extreme assumption - in this case perfect substitutes - may overturn the convergence result in the case of taxes.

Principles of Efficiency

- Absent distributive concerns, when the marginal social cost of emissions is constant across countries and there are no ancillary private benefits of abatement, a necessary condition for a given amount of abatement activity to be efficiently distributed is that the private marginal benefit of abatement (i.e. the price of emissions) is the same in each country.

- The first best level of emissions requires the additional condition that the marginal social costs and marginal social benefits of emissions are equal. When the marginal social cost of emissions is constant across countries and there are no ancillary private benefits of abatement, this implies that the price of emissions is equal to the marginal social cost of emissions.

Permits

- If permits are used the level of abatement is fixed and price convergence improves the efficiency of abatement activity. A number of mechanisms lead to convergence in permit prices across countries:

    1. Factor relocation and trade in goods
    2. Product market competition
    3. Permit trade

- Price convergence is likely to be most extensive when there is permit trade. Incentives to relocate to take advantage of emissions price differences will be limited by the costs involved in relocation. Similarly products may not be perfect substitutes and there may be costs of transporting goods across borders. Permit trade is therefore likely to increase the efficiency of the distribution of abatement even if both relocation and competition are possible.

- Furthermore if relocation is motivated by emissions price differences any costs incurred will be a deadweight loss. Similarly importing goods from competing producers in other countries (simply because of lower emissions prices) may also result in deadweight losses in the form of transportation costs.

Taxes

- If countries impose taxes then interactions between different countries can affect the overall level as well as the distribution of abatement. Producers are likely to opt for lower taxes wherever the rules make it possible. Therefore interactions such as factor relocation will tend to lead to lower average emissions prices and higher levels of emissions in total.

- The definition of the tax base will affect interactions between countries. If taxes are levied on final goods on the basis of the emissions they cause then there are no incentives for relocation or benefits to competitors in other countries. If taxes are levied on domestic emissions then relocation and competition are more likely. If tax bases overlap (e.g. if a firm is taxable both for emissions in country A and for goods sold in B) then governments must decide how to treat producers in this situation. Similarly governments must decide the extent to which permits from other countries can be used to reduce the tax liability of domestic producers.

- If relocation is costless or producers can freely choose whether to pay a tax in one country or buy permits from another, then (as long as some taxes are paid) the tax rate determines the price of emissions and the quantity of permits determines the allocation of revenues. The permit issuing country then has an incentive to increase the number of permits and to appropriate more of the revenues. This problem is likely to be less prominent if the two schemes are operated within regions of a single country or if there is coordination between countries about the allocation of revenues.

- Requiring producers to pay a tax on top of buying permits makes little real difference in the absence of permit trade (although the tax will act as a price floor for emissions rights in that country). If there is permit trade prices for permits will be equalized and producers will prefer to locate in the country with the lower additional tax rate.

Subsidies

- Governments can encourage emissions reductions through direct subsidies based on the amount of abatement or through indirect subsidies for the use of more efficient technologies. Direct subsidies for abatement can achieve the same outcomes as taxes in a single country except that revenues will now be negative. Indirect subsidies can achieve efficient relative pricing between technologies, but the overall level of prices is likely to be inefficiently low and will lead to excessive output. Both forms of subsidy are likely to attract firms relocating from other countries.

- In the short term subsidies for research into more efficient technologies will only affect the location and production of R&D, and there will be few interactions with other emissions policies. Subsidies for the adoption of new technologies are similar to other indirect subsidies.

Other Policy Instruments

- Regulation may impose firm-specific caps on emissions (equivalent to a scheme of non-tradable permits) or standards of efficiency for production. In both cases the shadow prices of emissions are likely to differ across firms let alone across countries, and the incentives to relocate will vary accordingly.

- If two countries impose different price ceilings and floors in a tradable permit scheme, the higher ceiling and the lower floor are redundant. A country that tries unilaterally to enforce a floor in a joint permit scheme may generate negative revenues.

- Border adjustments (e.g. rebates) can be used to mitigate the effects of policy differences. Import or export duties can narrow the gap between tax rates and will discourage firm relocation and competition from low tax countries. Such adjustments may be more difficult to reconcile with direct regulation or voluntary restrictions on emissions. For example export subsidies that encourage increased output may undermine any voluntary restrictions that are in place.

The Role of Coordination

Many of the more problematic aspects of cross-border interactions can be mitigated by effective coordination of different policies. As long as policies are mutually consistent undesirable outcomes – such as the unfair expropriation of revenues through issuing excessive quantities of permits – can be avoided.

Technical Appendix

A1. Preliminaries

Notation

C_i	Social/externality cost of output in country i
T_i	Tax in country i
Q_i	Quota of permits in country i
Q	Total quota of permits in both countries
$F_i(T_i)$	Demand/output in country i assuming no interactions with other countries
$P_i(Q_i)$	Inverse demand in country i given quota Q_i and no interactions
$P(Q)$	Aggregate inverse demand given quota level Q

Other Notes

- For ease of comparison demand is sometimes represented as the inverse of the inverse demand e.g. $P_b^{-1}(T_a)$.
- Assume $F_i(0) \rightarrow \infty$ so that $P_i(Q_i) > 0$ for any Q_i
- Trade in the context of taxes denotes tax regime choice for producers
- Factor/product mobility is here assumed to be either costless or infinitely costly

The First Best Condition

Suppose that the price of emissions is $\underline{P}_a$ in A and $\underline{P}_b$ in B. Total welfare comprises the private benefits of emissions and the social costs of emissions:

$$W = \int_0^{P_a^{-1}(\underline{P}_a)} P_a(t)\,dt + \int_0^{P_b^{-1}(\underline{P}_b)} P_b(t)\,dt - C_a P_a^{-1}(\underline{P}_a) - C_b P_b^{-1}(\underline{P}_b)$$

Differentiating with respect to $\underline{P}_a$, setting the first order condition to 0 and rearranging gives:[24]

$$\underline{P}_a = C_a$$

And differentiating with respect to $\underline{P}_b$ implies:

$$\underline{P}_b = C_b$$

The Principle of Equal Emissions Prices

Now suppose that the quantity of emissions is fixed so that $P_a^{-1}(\underline{P}_a) + P_b^{-1}(\underline{P}_b) = \underline{Q}$. Then conditional on the total level of emissions and assuming $C_a = C_b = C$ welfare can be expressed as:

$$W = \int_0^{P_a^{-1}(\underline{P}_a)} P_a(t)\,dt + \int_0^{\underline{Q} - P_a^{-1}(\underline{P}_a)} P_b(t)\,dt - C\underline{Q}$$

[24] Second order conditions for a maximum are omitted for clarity of exposition.

Differentiating with respect to $\underline{P}_a$ yields the first order condition:

$$0 = P_a(P_a^{-1}(\underline{P}_a))(dP_a^{-1}(\underline{P}_a)/ d\underline{P}_a) - P_b(Q - P_a^{-1}(\underline{P}_a))(dP_a^{-1}(\underline{P}_a)/ d\underline{P}_a)$$

$$= (\underline{P}_a - \underline{P}_b)(dP_a^{-1}(\underline{P}_a)/ d\underline{P}_a)$$

Which implies:

$\underline{P}_a = \underline{P}_b$

A2. Taxes only

Model 2.1 Benchmark tax case

$C_a = C_b = C$
$T_a < T_b$
No trade
No factor/product mobility
No border adjustments

Emissions/Output
A: $F_a(T_a)$
B: $F_b(T_b)$
Total: $F_a(T_a) + F_b(T_b)$

Revenues
A: $T_aF_a(T_a)$
B: $T_bF_b(T_b)$
Total: $T_aF_a(T_a) + T_bF_b(T_b)$

Production
First best implies $T_a = T_b = C$
$T_a \neq T_b$ implies inefficient production

Discussion
In this benchmark case countries are closed economies apart from having a collective desire to reduce emissions. Tax levels are set independently and local demand determines the level of output. Since tax levels differ (while the costs of emissions are the same) the outcome is productively inefficient.

Model 2.2 Tax regime choice

$C_a = C_b = C$
$T_a < T_b$
Trade
No factor/product mobility
No border adjustments

Emissions/Output
A: $F_a(T_a)$
B: $F_b(T_a)$

Total: $F_a(T_a) + F_b(T_a)$

Revenues

A: $T_aF_a(T_a) + T_aF_b(T_a)$

B: 0

Total: $T_aF_a(T_a) + T_aF_b(T_a)$

Production

First best iff $T_a = C$

Discussion

Since producers in B are now able to pay A's lower tax rate production and emissions in country B (and hence overall) increase relative to Model 2.1. B no longer receives any revenue from taxes illustrating the problem of tax competition between countries. Unlike Model 2.1 these results could be efficient if T_a is in fact the optimal tax rate.

Model 2.3 Factor/product mobility

$C_a = C_b = C$

$T_a < T_b$

No trade

Costless factor/product mobility

No border adjustments

Emissions/Output		Compared to Benchmark
A:	$F_a(T_a) + F_b(T_a)$	Higher
B:	0	Lower
Total:	$F_a(T_a) + F_b(T_a)$	Higher

Revenues

A: $T_aF_a(T_a) + T_aF_b(T_a)$

B: 0

Total: $T_aF_a(T_a) + T_aF_b(T_a)$

Production

First best iff $T_a = C$

Discussion

These results are similar to those of Model 2.2 except that all output now takes place in country A. If factor/product mobility is costless and there are no border adjustments then it makes no difference to total emissions whether firms can choose tax regimes or locations.

Model 2.4 Tax regime choice and factor/product mobility

$C_a = C_b = C$

$T_a < T_b$

Trade

Costless factor/product mobility

No border adjustments

Emissions/Output

A: between $F_a(T_a)$ and $F_a(T_a) + F_b(T_a)$

B: between 0 and $F_b(T_a)$

Total: $F_a(T_a) + F_b(T_a)$

Revenues

A: $T_aF_a(T_a) + T_aF_b(T_a)$

B: 0

Production

First best if and only if $T_a = C$

Discussion

Having both free choices of location and tax regimes does not affect total emissions or revenues. However there is a continuum of equilibria in which firms from B choose either to adopt A's tax regime while remaining in B or to relocate. Given even very small relocation costs $\varepsilon > 0$ the results would be identical to Model 2.2.

Model 2.5 Tax regime choice and border adjustments

$C_a = C_b = C$

$T_a < T_b$

Trade

No factor/product mobility

Border adjustments $\Delta \leq T_b - T_a$ imposed by B on emissions rights bought in A but used in B

Emissions/Output

A: $F_a(T_a)$

B: $F_b(T_a + \Delta)$

Total: $F_a(T_a) + F_b(T_a + \Delta)$

Revenues

Case 1: $\Delta < T_b - T_a$

A: $T_aF_a(T_a) + T_aF_b(T_a + \Delta)$

B: $\Delta F_b(T_a + \Delta)$

Case 2: $\Delta = T_b - T_a$

A: between $T_aF_a(T_a)$ and $T_aF_a(T_a) + T_aF_b(T_b)$

B: between $\Delta F_b(T_b)$ and $T_bF_b(T_b)$.

Production

First best requires $T_a = C$ and $\Delta = 0$

Discussion

Border adjustments increase B's revenues compared with Model 2.2. Output is somewhere between that of Model 2.1 and Model 2.2 because producers in B are paying a net tax rate between T_a and T_b. If T_a is optimal then any positive border adjustment is inefficient.

Model 2.6 Tax regime choice, border adjustments and mobility

$C_a = C_b = C$

$T_a < T_b$

Trade

Costless factor/product mobility

Border adjustments $\Delta \leq T_b - T_a$ imposed by B on all goods imported by relocating firms and on emissions rights bought in A but used in B

Similar to Model 2.5 but firms may choose to relocate. Given small costs of relocation the outcome is identical to Model 2.5.

A3. Subsidies

Elasticities

Two Technology Model

$F_a(T_a , k_{a,2}) = R_{a,1} Y_{a,1}(T_a , k_{a,2}) + R_{a,2} Y_{a,2}(T_a , k_{a,2})$

Where $R_{a,1} > R_{a,2}$ are the respective levels of emissions per unit output and $k_{a,2}$ denotes the cost of the cleaner technology.[25] Lowering $k_{a,2}$ through a subsidy lowers $Y_{a,1}$ and raises $Y_{a,2}$. Overall emissions will fall as long as:

$R_{a,1} (\partial Y_{a,1}/ \partial Y_{a,2}) (\partial Y_{a,2}/ \partial k_{a,2}) > - R_{a,2} (\partial Y_{a,2}/ \partial k_{a,2})$

Which implies:

$- R_{a,1} (\partial Y_{a,1}/ \partial Y_{a,2}) > R_{a,2}$

$(\partial Y_{a,1}/ \partial Y_{a,2}) (Y_{a,2}/ Y_{a,1}) = e_{12} > (R_{a,2} Y_{a,2}) / (R_{a,1} Y_{a,1})$

Variable Efficiency Model

$Y_a (\partial R_a/ \partial K_a) > - R_a (\partial Y_a/ \partial R_a) (\partial R_a/ \partial K_a)$

$1 > - (R_a/ Y_a) (\partial Y_a/ \partial R_a) = e_{YR}$

Compare this with the condition for taxes

$R_a (\partial Y_a/ \partial R_a) (\partial R_a/ \partial T_a) < - R_a (\partial Y_a/ \partial T_a) - Y_a (\partial R_a/ \partial T_a)$

$- R_a (\partial Y_a/ \partial R_a) < R_a (\partial Y_a/ \partial T_a) / (\partial R_a/ \partial T_a) + Y_a$

Which shows that the condition that the elasticity must satisfy is less strict in the case of taxes.

A4. Permits only

Model 4.1 Benchmark permit case

$C_a = C_b = C$
$Q_a + Q_b = Q$
No trade
No factor/product mobility
No border adjustments

Emissions/Output
A: Q_a

[25] The cost of the less clean technology is suppressed for notational clarity.

B: Q_b
Total: $Q = Q_a + Q_b$

Revenues
A: $Q_a P_a(Q_a)$
B: $Q_b P_b(Q_b)$
Total: $Q_a P_a(Q_a) + Q_b P_b(Q_b)$

Production
First best requires $P_a = P_b = C$

Model 4.2 Tradable permits

$C_a = C_b = C$
$Q_a + Q_b = Q$
Trade
No factor/product mobility
No border adjustments

Emissions/Output
A: $F_a(P(Q))$
B: $F_b(P(Q))$
Total: Q

Revenues
A: $Q_a P(Q)$
B: $Q_b P(Q)$
Total: $Q_a P(Q) + Q_b P(Q)$

Production
First best requires $P(Q) = C$ so $Q = P^{-1}(C)$

Discussion
Conditional on any given overall quantity cap it is efficient to have the same permit price in each country (because the marginal cost of abatement is then equalized). Full trading allows this to occur and unless individual country allowances are such that $P_a = P_b$ exactly in Model 4.1 trade improves efficiency. Total emissions are of course unchanged. Revenue comparisons depend on whether prices rise or fall when trade is allowed.

Model 4.3 Permits and factor/product mobility

$C_a = C_b = C$
$Q_a + Q_b = Q$
No trade
Costless factor/product mobility
No border adjustments

Essentially the same as Model 4.2 except that the output levels in each country may differ. In this case permits can't be traded so output is equal to the number of permits in each country.

Model 4.4 Tradable permits and factor/product mobility

$C_a = C_b = C$

$Q_a + Q_b = Q$
Trade
Costless factor/product mobility
No border adjustments

Similar to Models 2.2 and 2.3. The allocation of production could be the same as either Model 4.2 or 2.3 or anywhere in between. Very small costs of relocation $\varepsilon > 0$ would select the equilibrium identical to Model 4.2.

A5. Permits and taxes

Model 5.1 Benchmark permit and tax case

$C_a = C_b = C$
Q_b in country B
T_a in country A
No trade
No factor/product mobility
No border adjustments

Emissions/Output
A: $F_a(T_a)$
B: Q_b

Revenues
A: $T_aF_a(T_a)$
B: $Q_bP_b(Q_b)$

Production
First best requires $P_b(Q_b) = T_a = C$

Model 5.2 Trade of permits only

$C_a = C_b = C$
Q_b in country B
T_a in country A
Trade of permits only
No factor/product mobility
No border adjustments

Emissions/Output

Case 1: $T_a < P_b(Q_b)$
Demand for permits by producers in B is strong so that the price exceeds A's tax and no producers from A wish to buy permits.

A: $F_a(T_a)$
B: Q_b

Case2: $P_b(Q_b) < T_a < P_b(Q_b - F_a(T_a))$

Demand for permits in B alone is not so strong that the permit price is initially higher than A's tax. However not all of the producers in A who currently pay the tax would benefit from switching to permits since this would drive the price too high. Of producers in B there are ($Q_b - P_b^{-1}(T_a)$) who have values below T_a and these are displaced by some of the producers in A. NB in equilibrium with $P_b = T_a$ the permits must be used up first.

A: $F_a(T_a) - (Q_b - P_b^{-1}(T_a))$
B: Q_b

Overall demand falls relative to Model 5.1 since $Q_b > P_b^{-1}(T_a)$

Case 3: $P_b(Q_b - F_a(T_a)) < T_a$
A: 0
B: Q_b

Revenues

Case 1: $T_a < P_b(Q_b)$
A: $T_aF_a(T_a)$
B: $Q_bP_b(Q_b)$

Case2: $P_b(Q_b) < T_a < P_b(Q_b - F_a(T_a))$
A: $T_a(F_a(T_a) - (Q_b - P_b^{-1}(T_a)))$
B: T_aQ_b
Overall revenue falls relative to Model 5.1 since $Q_b > P_b^{-1}(T_a)$

Case 3: $P_b(Q_b - F_a(T_a)) < T_a$
A: 0
B: $Q_bP_b(Q_b - F_a(P_b))$

Production
First best requires $P_b = T_a = C$ when there is production in both countries

Model 5.3 Trade of permits and tax regime choice

$C_a = C_b = C$
Q_b in country B
T_a in country A
Trade of permits and taxes
No factor/product mobility
No border adjustments

Emissions/Output

Case 1: $T_a < P_b(Q_b)$
A: $F_a(T_a) + P_b^{-1}(T_a) - Q_b$
B: Q_b

Case 2: $P_b(Q_b) < T_a < P_b(Q_b - F_a(T_a))$
A: $F_a(T_a) - (Q_b - P_b^{-1}(T_a)))$
B: Q_b

Case 3: $P_b(Q_b - F_a(T_a)) < T_a$

A: 0
B: Q_b

Revenues

Case 1: $T_a < P_b(Q_b)$
A: $T_a (F_a(T_a) + P_b^{-1}(T_a) - Q_b)$
B: T_aQ_b

Case 2: $P_b(Q_b) < T_a < P_b(Q_b - F_a(T_a))$
A: $T_a(F_a(T_a) - (Q_b - P_b^{-1}(T_a)))$
B: T_aQ_b

Case 3: $P_b(Q_b - F_a(T_a)) < T_a$
A: 0
B: $Q_bP_b(Q_b - F_a(P_b))$

Production
First best requires $P_b = T_a = C$

Model 5.4 Factor/product mobility

$C_a = C_b = C$
Q_b permits issued in country B
Tax T_a in country A
No trade
Costless factor/product mobility
No border adjustments

Similar to Model 5.3 except that outcomes are achieved by firm relocation.

Model 5.5 Trade of permits and border adjustments

$C_a = C_b = C$
Q_b in country B
T_a in country A
Trade of permits only
No factor/product mobility
Border adjustments of $\Delta \leq T_a - P_b$ imposed by A on permits that are traded across borders

Emissions/Output

Case 1: $T_a < P_b(Q_b) + \Delta$
A: $F_a(T_a)$
B: Q_b

Case 2: $P_b(Q_b) + \Delta < T_a < P_b(Q_b - F_a(T_a)) + \Delta$
Price in B would equal $T_a - \Delta$. The producers in B with values below $T_a - \Delta$ would be displaced. There are $Q_b - P_b^{-1}(T_a - \Delta)$ of these.
A: $F_a(T_a) - (Q_b - P_b^{-1}(T_a - \Delta))$
B: Q_b

Case 3: $P_b(Q_b - F_a(T_a)) + \Delta < T_a$
A: 0
B: Q_b

Revenues

Case 1: $T_a < P_b(Q_b) + \Delta$
A: $T_a F_a(T_a)$
B: $Q_b P_b(Q_b)$

Case 2: $P_b(Q_b) + \Delta < T_a < P_b(Q_b - F_a(T_a)) + \Delta$
Price in B would equal $T_a - \Delta$. The producers in B with values below $T_a - \Delta$ would be displaced. There are $Q_b - P_b^{-1}(T_a - \Delta)$ of these.
A: $T_a(F_a(T_a) - (Q_b - P_b^{-1}(T_a - \Delta))) + \Delta(Q_b - P_b^{-1}(T_a - \Delta))$
B: $Q_b(T_a - \Delta)$

Case 3: $P_b(Q_b - F_a(T_a)) + \Delta < T_a$
A: 0
B: $Q_b P_b(Q_b - F_a(P_b + \Delta))$

Production
Inefficient allocation of abatement unless prices are equal (which only occurs in Case 3).

Model 5.6 Tradable permits, tax regime choice and border adjustments

$C_a = C_b = C$
Q_b permits issued in country B
Tax T_a in country A
Trade of permits and taxes
No factor/product mobility
Border adjustments of $\Delta \leq T_a - P_b$ imposed by A on permits that are traded across borders

Emissions/Output

Case 1: $T_a < P_b(Q_b) + \Delta$
A: $F_a(T_a) + P_b^{-1}(T_a) - Q_b$
B: Q_b

Case 2: $P_b(Q_b) + \Delta < T_a < P_b(Q_b - F_a(T_a)) + \Delta$
Price in B would equal $T_a - \Delta$. The producers in B with values below $T_a - \Delta$ would be displaced. There are $Q_b - P_b^{-1}(T_a - \Delta)$ of these.
A: $F_a(T_a) - (Q_b - P_b^{-1}(T_a - \Delta))$
B: Q_b

Case 3: $P_b(Q_b - F_a(T_a)) + \Delta < T_a$
A: 0
B: Q_b

Revenues

Case 1: $T_a < P_b(Q_b) + \Delta$
A: $T_a(F_a(T_a) + P_b^{-1}(T_a) - Q_b)$
B: $Q_b T_a$

Case 2: $P_b(Q_b) + \Delta < T_a < P_b(Q_b - F_a(T_a)) + \Delta$
Price in B would equal $T_a - \Delta$. The producers in B with values below $T_a - \Delta$ would be displaced. There are $Q_b - P_b^{-1}(T_a - \Delta)$ of these.

A: $T_a(F_a(T_a) - (Q_b - P_b^{-1}(T_a - \Delta))) + \Delta(Q_b - P_b^{-1}(T_a - \Delta))$
B: $Q_b(T_a - \Delta)$

Case 3: $P_b(Q_b - F_a(T_a)) + \Delta < T_a$
A: 0
B: $Q_b P_b(Q_b - F_a(P_b + \Delta))$

Production
Inefficient allocation of abatement unless prices are equal (which only occurs in Case 3).

A6. Ceilings and Floors

Model 6.1 Benchmark case

$C_a = C_b = C$
Q_a in country A subject to bounds $P_{a,h} \geq P_a(Q_a) \geq P_{a,l}$
Q_b in country B
No trade
No factor mobility
No border adjustments

Emissions

A:	Q_a	if $P_{a,h} \geq P_a(Q_a) \geq P_{a,l}$
	$P_a^{-1}(P_{a,l})$	if $P_a(Q_a) < P_{a,l}$
	$P_a^{-1}(P_{a,h})$	if $P_{a,h} < P_a(Q_a)$
B:	Q_b	

Revenues

A:	$Q_a P_a(Q_a)$	if $P_{a,h} \geq P_a(Q_a) \geq P_{a,l}$
	$P_a^{-1}(P_{a,l})P_{a,l}$	if $P_a(Q_a) < P_{a,l}$
	$P_a^{-1}(P_{a,h})P_{a,h}$	if $P_{a,h} < P_a(Q_a)$
B:	$Q_b P_b(Q_b)$	

Production
Efficiency would require price equalization.

Discussion
The existence of a price collar in country A limits the range of values for permit prices in A but necessarily means that the price cap is not always adhered to.

Model 6.2 Single price collar with trade

$C_a = C_b = C$
Q_a in country A subject to bounds $P_{a,h} \geq P_a \geq P_{a,l}$
Q_b in country B
Trade
No factor mobility

No border adjustments

Emissions

Total: $Q_a + Q_b$ if $P_{a,h} \geq P(Q) \geq P_{a,l}$

$P^{-1}(P_{a,l})$ if $P(Q) < P_{a,l}$

$P^{-1}(P_{a,h})$ if $P_{a,h} < P(Q)$

Revenues

A: $Q_aP(Q)$ if $P_{a,h} \geq P(Q) \geq P_{a,l}$

$(P^{-1}(P_{a,l}) - Q_b)P_{a,l}$ if $P(Q) < P_{a,l}$

$(P^{-1}(P_{a,h}) - Q_b)P_{a,h}$ if $P_{a,h} < P(Q)$

B: $Q_bP(Q)$ if $P_{a,h} \geq P(Q) \geq P_{a,l}$

$Q_bP_{a,l}$ if $P(Q) < P_{a,l}$

$Q_bP_{a,h}$ if $P_{a,h} < P(Q)$

Production

Single price means that conditional on the price and cap set the production decisions will be efficient.

Discussion

The presence of a single collar imposed in A means that the price of B's permits is similarly affected.

Model 6.3 Two price collars with trade

$C_a = C_b = C$

Q_a in country A subject to bounds $P_{a,h} \geq P_a \geq P_{a,l}$

Q_b in country B subject to bounds $P_{b,h} \geq P_b \geq P_{b,l}$

Trade

No factor mobility

No border adjustments

Emissions

Total: $Q_a + Q_b$ if $\min\{P_{a,h}, P_{b,h}\} \geq P(Q) \geq \max\{P_{a,l}, P_{b,l}\}$

$P^{-1}(\max\{P_{a,l}, P_{b,l}\})$ if $P(Q) < \max\{P_{a,l}, P_{b,l}\}$

$P^{-1}(\min\{P_{a,h}, P_{b,h}\})$ if $\min\{P_{a,h}, P_{b,h}\} < P(Q)$

Revenues

A: $Q_aP(Q)$ if $\min\{P_{a,h}, P_{b,h}\} \geq P(Q) \geq \max\{P_{a,l}, P_{b,l}\}$

$(P^{-1}(\max\{P_{a,l}, P_{b,l}\}) - Q_b) \max\{P_{a,l}, P_{b,l}\}$ if $P(Q) < \max\{P_{a,l}, P_{b,l}\}$

$(P^{-1}(\min\{P_{a,h}, P_{b,h}\}) - Q_b)\min\{P_{a,h}, P_{b,h}\}$ if $\min\{P_{a,h}, P_{b,h}\} < P(Q)$

B: $Q_bP(Q)$ if $\min\{P_{a,h}, P_{b,h}\} \geq P(Q) \geq \max\{P_{a,l}, P_{b,l}\}$

$Q_b\max\{P_{a,l}, P_{b,l}\}$ if $P(Q) < \max\{P_{a,l}, P_{b,l}\})$

$Q_b\min\{P_{a,h}, P_{b,h}\}$ if $\min\{P_{a,h}, P_{b,h}\} < P(Q)$

Production

Single price means that conditional on the price and cap set the production decisions will be efficient.

Discussion

The tighter ceiling and floor will determine the aggregate price collar when there is trade. Hence Model 6.2 above can be viewed as a special case of Model 6.3.

A7. Further Observations

Model 7.1 Benchmark simultaneous taxes and permits

$C_a = C_b = C$
$Q_a + Q_b = Q$
Tax T_a also imposed by A
$P_a(Q_a , T_a)$ is price of permits in A given tax T_a
No trade
No factor/product mobility
No border adjustments

Emissions/Output

A:	Q_a	if $P_a(Q_a , T_a) > 0$
	$F_a(T_a)$	if $P_a(Q_a , T_a) = 0$
B:	Q_b	

Revenues

A:	$Q_a (P_a(Q_a , T_a) + T_a)$	if $P_a(Q_a , T_a) > 0$
	$T_aF_a(T_a)$	if $P_a(Q_a , T_a) = 0$
B:	$Q_bP_b(Q_b)$	

Production
First best requires $P_a = P_b = C$

Discussion
$P_a(Q_a , T_a) = \max\{P_a(Q_a , 0) - T_a , 0\}$
where $P_a(Q_a , 0)$ is the price of permits in the absence of the extra tax
$P_a(Q_a , T_a) > 0$ is equivalent to $P_a(Q_a , 0) > T_a$

Model 7.2 Permit trade

$C_a = C_b = C$
$Q_a + Q_b = Q$
Tax T_a also imposed by A
$P(Q , T_a)$ is overall price of permits given tax T_a
Permit trade only
No factor/product mobility
No border adjustments

Emissions/Output

A:	$F_a(P(Q , T_a) + T_a)$
B:	$F_b(P(Q , T_a)$
Total:	$Q = Q_a + Q_b$

Revenues

A:	$T_a F_a(P(Q , T_a) + T_a) + Q_a P(Q , T_a)$
B:	$Q_b P(Q , T_a)$

Production
First best requires $P_a = P_b = C$
However $P_a = P(Q , T_a) + T_a$ and $P_b = P(Q , T_a)$ so inefficient

Discussion
Because of the assumption that demand is very large for free permits the market permit price is bounded away from 0.

Model 7.3 Factor Mobility

$C_a = C_b = C$
$Q_a + Q_b = Q$
Tax T_a also imposed by A
$P_a(Q_a , T_a)$ is price of permits in A given tax T_a
No trade
Costless factor/product mobility
No border adjustments

Emissions/Output

A:	Q_a	if $P_a(Q_a , T_a) > 0$
	$F_a(T_a)$	if $P_a(Q_a , T_a) = 0$
	0	if $P(Q_b) < T_a$
B:	Q_b	

Revenues

A:	$Q_a (P_a(Q_a , T_a) + T_a)$	if $P_a(Q_a , T_a) > 0$
	$T_aF_a(T_a)$	if $P_a(Q_a , T_a) = 0$
	0	if $P(Q_b) < T_a$
B:	$Q_bP_b(Q_b)$	

Production
First best requires $P_a = P_b = C$
Net price equalization makes this a possibility.

Discussion
Net price equalization $P_a(Q_a , T_a) + T_a = P_b(Q_b)$
Where $P_a(Q_a , T_a) = \max\{P_a(Q_a , 0) - T_a , 0\}$
Similar to a price floor of $\min\{P(Q_b) , T_a\}$