Department of Trade and Industry

The Noise Emission in the Environment by Equipment for Use Outdoors Directive 2000/14/EC

A Guide for Manufacturers to the Evaluation of Uncertainties

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CONTENTS

This guide is intended to offer assistance to manufacturers of outdoor equipment to understand the provisions for measurement uncertainty in the Directive. It is not an authoritative interpretation of the Directive, which is a matter for the Courts.

The guide seeks to explain the provisions of the Directive for measurement uncertainty only and does not attempt to address all requirements of the Directive. You should refer to the Directive itself (2000/14/EC) for a full statement of the requirements.

1. INTRODUCTION

The EC Directive relating to the noise emission in the environment by equipment for use outdoors, 2000/14/EC, requires equipment covered by the Directive to be labelled with a guaranteed sound power level. Article 3(f) of the Directive defines the guaranteed sound power level as a sound power level that includes an allowance for uncertainties in the determination of sound power level due to production variation and measurement procedures. For one-off pieces equipment uncertainties due to measurement procedures only apply.

In general terms, the uncertainty of a measurement tells us something about the quality of the measurement and may be described as the doubt that exists about the result of the measurement. Unlike an error, which is the difference between a measured value and the true value, an uncertainty is a quantification of the doubt about a measured result. The reason for introducing a guaranteed sound power level in the Directive 2000/14/EC was to ensure that manufacturers take account of these uncertainties particularly in relation to equipment which is subject to noise limits.

The aim of this guide is to help manufacturers determine the guaranteed sound power level for the equipment subject to the Directive. It offers a method by which manufacturers may calculate measurement and production uncertainties and how they may decide what **correction should be added** to the measured sound power level of a piece of equipment to give the guaranteed level. Established statistical methods are used for this purpose.

Since there is always a margin of doubt about any measurement, we need to ask, “How big is the margin?” and “How bad is the doubt?” Thus two terms need to be considered in order to quantify an uncertainty. One is the width of the margin, or **interval**. The other is a **confidence level**, and states how sure we are that the true value is within the margin. These terms are also described in this guide.

Manufacturers, however, are not bound to follow the procedures outlined in this guide. The Directive does not stipulate *how* a manufacturer should assess uncertainties in the determination of the sound power level only that he has to. The manufacturer is therefore free to use other methods.

This guide deals with two sources of uncertainty:

- repeatability uncertainty - i.e. uncertainties which result from variations in sound power level which arise in the measurement process (taken from one machine),

- production uncertainty - i.e. uncertainties which result from variations in sound power level which arise in the production process (taken from different machines of the same type).

Methods of obtaining estimates of these two uncertainties are given with worked examples in the pages that follow.

This guide does not deal with reproducibility uncertainties (i.e. uncertainties which result from variations in sound power level which arise in the measurement process – taken from the same machine but at different times, under different conditions (different laboratory, operator and apparatus)). For the purposes of this guide reproducibility uncertainties are not considered the responsibility of manufacturers but those persons responsible for checking or verifying manufacturers' equipment such as Notified Bodies or an enforcement authority. However, reproducibility uncertainties are dealt with in ISO 4871, which is referred to in the 'Using International standards' section of this guide. Manufacturers may use this standard if they wish.

2. UNCERTAINTIES DUE TO MEASUREMENT PROCEDURE

The measurement procedures uncertainty required by the Directive is the **repeatability uncertainty** which is a measure of the closeness of agreement between mutually independent sound power level determinations obtained under repeatability conditions. These are where mutually independent sound power level determinations are obtained using the same method on a single machine at the same measurement site by the same operator(s) using the same equipment within a short interval of time.

The quantity that needs to be obtained is the **standard deviation** of repeatability uncertainty, this is referred to as σ_r. This is a parameter of dispersion of the distribution of the sound power level determinations under repeatability conditions and tells us how different, in a set of sound power level determinations, individual sound power level determinations typically are from the average of the set.

The "true" value of σ_r can only be found from a very large (infinite) set of sound power level determinations. However, statistics enable an estimate, S_r of the standard deviation to be calculated from a smaller number, "n" of sound power level determinations. This estimate, S_r can be obtained using the expression:

$$S_r = \sqrt{\frac{\sum_{i=1}^{i=n}\left(x_i - \overline{x}\right)^2}{n-1}}$$

where, n is the number of repeated sound power level determinations
x_i is the sound power level determination of the ith repeat determination
$\overline{x}$ is the arithmetic mean of the n repeat determinations.

A worked example is given in EXAMPLE 1.

The more repeat determinations "n" that are made the better the estimate (S_r) will be. However, carrying out more sound power level determinations takes a considerable effort and yields diminishing returns. The choice of "n" is discussed in Section 6 where it is concluded that for practical purposes a minimum of five repeat measurements should be used.

For the purposes of the Directive it is suggested that a minimum of five repeat sound power level determinations should be made.[1]

In general, S_r will be small. It is possible that a value of S_r for one type of machine may be assumed to be the same as that obtained for another similar type of machine, thus reducing the noise measurement effort.

3. UNCERTAINTIES DUE TO PRODUCTION VARIATION

The process here is exactly the same as that outlined above for the calculation of the standard deviation of repeatability uncertainty but in this case a single sound power level determination is carried out on a number of machines taken from the production line. Again the quantity to be determined is the standard deviation, but this time it is the standard deviation of production uncertainty $\sigma_{p.}$ Again a true value of σ_p, can only be obtained from a very large set of sound power level determinations. However, an estimate, S_p of the standard deviation of production uncertainty can be obtained from:

$$S_p = \sqrt{\frac{\sum_{i=1}^{i=n}\left(x_i - \overline{x}\right)^2}{n-1}}$$

where, n is the number of machines
x_i is the sound power level determination of the ith machine
$\overline{x}$ is the arithmetic average of the sound power level determinations of all n machines.

The more machines that are used the better the estimate (S_p) will be. However, carrying out sound power level determinations on a number "n" of machines takes a considerable effort and yields diminishing returns as the number of machines is increased. The choice of "n" is discussed in Section 6 where it is concluded that for practical purposes a minimum of five machines should be used.

For the purposes of the Directive it is suggested that sound power level determinations should be made on a minimum of five machines.[2]

For a production line process that has just begun, it is possible that, initially, the value of S_p may be assumed to be the same as that obtained for a similar machine on a similar production line that has the same level of quality control. **However, the**

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[1] This is a suggested value only, ultimately it is the responsibility of the manufacturer to decide upon the value of "n".
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sampling procedure should continue during the lifetime of the production line in order to ensure that this assumed value of S_p is a reasonable estimate and that its value is maintained.

4. DETERMINING THE TOTAL STANDARD DEVIATION

In order to obtain the total estimate of the uncertainty associated with a sound power level determination, it is necessary to add the standard deviation for repeatability and production. The estimated total standard deviation is written as S_t and in this case for the purposes of the Directive will be obtained from the summation of S_r and S_p in quadrature using:

$$S_t = \sqrt{{S_r}^2 + {S_p}^2}$$

A worked example is given in EXAMPLE 2

EXAMPLE 1

CALCULATION OF A STANDARD DEVIATION OF UNCERTAINTY

In the Table below x_1, x_2, x_3, x_4 and x_5 can represent **either** five repeat sound power level determinations on a single machine for the purposes of calculating S_r, or single sound power level determinations carried out on five machines for the purposes of calculating S_p.

repeat or machine	sound power level determination (dB)	difference $(x_i - \bar{x})$	difference squared $(x_i - \bar{x})^2$
x_1	88.70	0.58	0.3364
x_2	87.90	- 0.22	0.0484
x_3	88.20	0.08	0.0064
x_4	87.80	- 0.32	0.1024
x_5	88.00	- 0.12	0.0144

The standard deviation is evaluated as follows.

Start by finding the arithmetic average

$$\bar{x} = (88.70 + 87.90 + 88.20 + 87.80 + 88.00) \div 5$$
$$= 440.6 \div 5$$

so the **average is 88.12 dB**

Next find the difference between each sound power level determination and the average $(x_i - \bar{x})$, listed in column 3.

Then square each of these differences $(x_i - \bar{x})^2$, listed in column 4.

Next find the total of the five numbers in column 4 and divide by n - 1
(in this case n = 5 so, n - 1 is 4)

$$\Sigma (x_i - \bar{x})^2 \div 4 = (0.3364 + 0.0484 + 0.0064 + 0.1024 + 0.0144) \div 4$$
$$= 0.508 \div 4$$
$$= 0.127$$

The estimated standard deviation, S is found by taking the square root

Or, $S = \sqrt{0.127}$

So, $\mathbf{S = 0.36}$ **dB** (rounded to two decimal places)

EXAMPLE 2

CALCULATION OF A TOTAL STANDARD DEVIATION

If we have set of machines and measurement process that results in, say:

S_r = 0.20 dB and S_p = 0.36 dB

The total standard deviation S_t is given by:

$$S_t = \sqrt{S_r{}^2 + S_p{}^2}$$

So,

$$S_t = \sqrt{0.2^2 + 0.36^2} \quad = \quad \sqrt{0.04 + 0.1296} \quad = \quad \sqrt{0.1696}$$

Therefore, $\boldsymbol{S_t =}$ **0.41 dB** (rounded to two decimal places)

5. ASSESSING THE VALUE OF THE CORRECTION, K

The Directive requires that a machine is labelled with a guaranteed sound power level, which must take account of measurement and production uncertainties. The previous examples in this guide have shown how both these quantities of uncertainty can be estimated. We now need to consider the **correction, K** that is to be added to the average sound power determination to give us a guaranteed level.

A standard deviation is a measure of the spread of a set of sound power level determinations, describing how values typically differ from the average of the set. It is possible using a set of tables associated with a statistical distribution known as **Student's t**, to estimate the possibility of a single sound power level determination being greater than the average sound power level by a given amount. It is usual to express this as a percentage of sound power level determinations that are expected to be less than a given sound power level. This percentage is called a **confidence level** and is an indication of how confident you are of the sound power level determination of a particular machine not exceeding a given sound power level.

For the purposes of the Directive it is suggested that a confidence level of 95% is adopted.[3]

For instance for the case where the number of sound power level determinations in a set were very large it can be stated that 95% of sound power level determinations will be less than the average sound power level plus 1.645 times the standard deviation. The constant, 1.645 is called the **coverage factor** and is dependent on the value of "n" and on the confidence level (the percentage) chosen.

For the recommended confidence level of 95% and value of n of 5, the coverage factor is 2.132

An extract from the table describing the Student's t distribution, presented in a form suitable for this guide, is given in Section 6.

A worked example of assessing the value of K is given in EXAMPLE 3.

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[3] This is a suggested value only, ultimately it is the responsibility of the manufacturer to decide upon the confidence interval.

EXAMPLE 3

CALCULATION OF THE CORRECTION, K

Using the data from the EXAMPLE 2, where a standard deviation, S_t of 0.41 dB was obtained using five repeat measurements and five machines with an average sound power level of 88.12 dB, the value of the coverage factor taken from the Student's t statistical table assuming a confidence level of 95% is 2.132.

This will result in a value of K of, $2.132 * 0.41$, or **0.874 dB**

Thus it can be assumed that 95% of sound power level determinations made on this particular type of machine will be less than the average value plus K, so:

the labelled value = 88.12 + 0.874

= 88.99 dB or, **89 dB** (rounded to the nearest 1 dB)

6. THE CHOICE OF A VALUE FOR "n"

The estimated value of a standard deviation will become closer to the "true" value as the number, n, of repeat measurements carried out or the number of machines used is increased. To assess a practical value for n the magnitude of the coverage factor should be considered. The value of the coverage factor is obtained from the table associated with the Student's t distribution, an extract **for a 95% confidence level**, presented in a form suitable for this guide, is given below together with the **correction, K that would be applied assuming the data in the examples (where S_t = 0.41 dB).**

N	coverage factor	correction, K (dB)
2	6.314	2.59
3	2.920	1.20
4	2.353	0.96
5	2.132	0.87
6	2.015	0.83
7	1.943	0.80
8	1.895	0.78
9	1.860	0.76
10	1.833	0.75
15	1.761	0.72
20	1.729	0.71
∞	1.645	0.67

It can be seen that for values of n greater than about 5 the change in the value of K is relatively small. For example, increasing the value of n from 5 to 20 will only reduce the value of K from 0.87 dB to 0.71 dB, less than 0.2 dB. It is therefore recommended that for practical purposes a minimum of five repeat measurements or five machines should be used.

7. USING INTERNATIONAL STANDARDS

If values for S_r and S_p are not available, then the guidance given in ISO 4871:1997 may be followed. In this standard the guaranteed sound power level is referred to as the declared sound power level.

For a single machine (subject to unit verification) a value for K is obtained from:

$$K = 1.645 \text{ times } \sigma_R$$

where, 1.645 is the coverage factor
σ_R is the standard deviation of **reproducibility**
Here it is a standard deviation (σ_R not S_R) that is used and so the coverage factor for a 95% confidence level is selected assuming n is infinite.

For engineering grade accuracy the value of σ_R is 1.5 dB, so:

$$K = 1.645 \text{ times } 1.5 = 2.47 \text{ dB}$$

For survey grade accuracy the value of σ_R is 3 dB, so:

$$K = 1.645 \text{ times } 3 = 4.94 \text{ dB}$$

When considering a batch of machines the standard gives an example using three machines and here a value for K is obtained from:

$$K = 1.5 \text{ times } \sigma_M$$

where, 1.5 is a constant derived from sampling theory considerations[4]
σ_M is the **reference standard deviation**

For engineering grade accuracy the value of σ_M is 2.5 dB, so:

$$K = 1.5 \text{ times } 2.5 = 3.75 \text{ dB}$$

For survey grade accuracy the value of σ_M is 4 dB, so:

$$K = 1.5 \text{ times } 4 = 6.00 \text{ dB}$$

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[4] For more detail on the calculations above see, International Organisation for Standardisation, ISO 4871:1997, Acoustics – Declaration and verification of noise emission values of machinery and equipment.

8. THE TERMINOLOGY

The glossary below gives a brief explanation of a few statistical terms.

Confidence level
number (e.g. 95%) expressing the degree of confidence in a result.

Correction K
number added to the measured result to give a guaranteed sound power level for a particular level of confidence.

Coverage factor
number which is multiplied by the total standard deviation to obtain the correction, K.

Estimated standard deviation (*S*)
an estimate of the standard deviation of the "population" based on a limited sample.

Interval (confidence interval)
margin within which the true value being measured can be said to lie, with a given level of confidence.

Reference standard deviation (σ_M)
A total standard deviation specified for the family of machinery and equipment which is considered typical for batches of equipment from this family

Repeatability (standard deviation of) **(σ_r)**
closeness of the agreement between repeated sound power determinations on the same machine under the same conditions.

Reproducibility (standard deviation of) **(σ_R)**
standard deviation obtained under reproducibility conditions; that is, repeated sound power level determinations carried out on the same machine at different times and under different conditions (different laboratory, operator and apparatus). The standard deviation of reproducibility therefore includes the standard deviation of repeatability.

Standard deviation (σ)
a measure of the spread of a set of results, describing how values typically differ from the average of a set. Where it is not possible to obtain an infinite set of results (in practice it never is) we instead use the estimated standard deviation.

Student's t
a statistical distribution defined by, the ratio of the deviation of the mean of a sample of size n from an expected value, to its standard deviation.

For more rigorous definitions see for example:

BIPM, IEC, IFCC, IUPAC, OIML. *Guide to the expression of uncertainty in measurement*. International Organisation for standardisation, Geneva, Switzerland. ISBN 92-67-10188-9, First Edition 1993, corrected and reprinted 1995. (BSI Equivalent: BSI PD 6461:1995, Vocabulary of Metrology, Part 3. *Guide to the expression of uncertainty in measurement.* BSI ISBN 0 580 23482 7.)

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