

TECHNICAL APPENDIX 2: THE TREATMENT OF LOCAL ACCESSIBILITY INDICATORS IN ACCESSION

INTRODUCTION

1. The purpose of this document is to provide information for the contractor undertaking the development of the accessibility planning software tool, as well as providing general information for those interested in the accessibility indicators and measures underpinning the accessibility planning software tool.
2. The software tool will enable users to undertake a range of accessibility assessments undertaken using each of the following measures of accessibility:
 - Access measures.
 - Network measures.
 - Continuous measures.

ACCESS MEASURES

3. Essentially define the ease of access to/from the transport network. Usually applied to public transport networks.

ACCESS TO PUBLIC TRANSPORT INDICATORS:

4. Considering for example one of the potential indicators outlined in Annex A of the Social Exclusion Unit (SEU) final report on transport and social exclusion (main report) and the current rural bus accessibility indicator respectively:
 - Proportion of people within 10 minutes walk of a 5, 10, 15 minute bus service.
 - Proportion of households in rural areas within a 10-13 minute walk of an hourly or better bus service.
5. This generalises to access to/from the public transport network defined in terms of journey time/distance to/from an access point with user defined service frequencies for different user defined time periods and days of the week.
6. The software tool will enable users to define public transport mode, walk mode time/distance threshold C, public transport service frequency level, time of day and day of week which will be used to undertake the accessibility assessment. As an example:
 - i. Public transport mode: Bus
 - ii. Vehicle type: Low floor bus
 - iii. Walk time: 10 minutes
 - iv. Public transport service frequency level: At least 4 per hour
 - v. Day of week: Monday-Friday

- vi. Time of day: AM peak
7. Access points satisfying the above criteria (i, ii, iii, iv, v, vi) are then identified and these are then treated as destinations for the subsequent walk routing process. Journey time/distance contours are subsequently produced (preceded by an interpolation stage). Accessibility reports and queries are separately to be used to determine the population groupings present within the defined journey time/distance threshold C . The mathematical definition of the population within this threshold is:

$$Q(R^s, B, C) = \sum_{i=1}^I R_i^s \times \partial_i^{BC} \quad (1)$$

Where,

$Q(R^s, B, C)$ The total resident population in segment s which lives within the travel threshold C of the nearest access point of type B . Indicator range $0 - \sum_{i=1}^I R_i^s$ inclusive.

R_i^s The number of residents in segment s living in zone i .

∂_i^{BC} An indicator function which assumes the following values:
 1 - Unity if zone i is within the travel threshold C of at least one access point of type B .
 0 - Zero otherwise.

8. The aforementioned expression represents a destination based threshold (cumulative opportunity) accessibility measure, in which the target destinations are the access points which satisfy the specified criteria, and these access points are assumed to be perfect substitutes for each other.
9. The proportion of the total population within the access-to-public-transport threshold (as proposed in paragraph 4) is:

$$q(R^s, B, C) = \frac{Q(R^s, B, C)}{\sum_{i=1}^I R_i^s} = \frac{\sum_{i=1}^I R_i^s \times \partial_i^{BC}}{\sum_{i=1}^I R_i^s} \quad (2)$$

Where,

$q(R^s, B, C)$ The proportion of the resident population in segment s which lives within the travel threshold C of the nearest access point of type B . Indicator range $0 - 1$ inclusive.

10. It should be noted that equations 1 and 2 can be applied without an intermediate interpolation and contouring phase with the actual summary indicators calculated through application of equation 1 to each origin point location in turn. It however should be noted that this will most probably produce a result which is not consistent with the use of a GIS based summation phase (i.e. aerial based proportional sum, point count or otherwise).

PTALS PLUS INDEX:

11. The Public Transport Accessibility Levels (PTALs) Plus index will also be incorporated within the software tool for consistency. The Transport for London (TfL) methodology for defining the PTALs Plus index is presented in the annex to this paper.

NETWORK MEASURES

12. These indicators define the ease of access through the transport network to/from a particular service in terms of journey time, distance, cost or generalised cost, for different departure times or time periods and days of the week.

13. The software tool will facilitate multi-modal network accessibility assessments through the transport network to/from home and activity locations. Users will be able to specify the mode(s) of travel, the departure time or the time period, the day of week, the origin locations, and service locations. As an example consider:

- i. Modes of travel: Public transport fixed route / Walk
- ii. Departure time: 08:00 hours or am peak
- iii. Day of week: Monday-Friday
- iv. Origin locations: Home address points/grid
- v. Destination locations: Secondary schools

14. Network accessibility assessment is then undertaken utilising transport and land use/service data items satisfying the above criteria (i, ii, iii, iv, v). Journey time/distance/cost/generalised contours are subsequently produced (preceded by an interpolation stage).

15. Accessibility reports and queries can be used to determine the total resident population type s located within the defined journey time/distance/cost/generalised threshold C of the nearest activity location of type K (equation 3).

Where,

$$P(R^s, K, C) = \sum_{i=1}^I R_i^s \times \partial_i^{KC} \tag{3}$$

$P(R^s, K, C)$ The total resident population in segment s which lives

within the travel threshold C of the nearest activity location of type K . Indicator range $0 - \sum_{i=1}^I R_i^s$ inclusive.

∂_i^{KC}

An indicator function which assumes the following values:
 1 - Unity if zone i is within the travel threshold C of at least one activity location of type K .
 0 - Zero otherwise.

16. An additional formulation of this expression is the proportion of the relevant population type s located within the defined journey time/distance/cost/generalised threshold C of the nearest activity location of type K (equation 3):

$$p(R^s, K, C) = \frac{P(R^s, K, C)}{\sum_{i=1}^I R_i^s} = \frac{\sum_{i=1}^I R_i^s \times \partial_i^{KC}}{\sum_{i=1}^I R_i^s} \quad (4)$$

Where,

$p(R^s, K, C)$

The proportion of the resident population in segment s which lives within the travel threshold C of the nearest activity location of type K . Indicators range $0 - 1$ inclusive.

17. The aforementioned expressions represent a destination based threshold (cumulative opportunity) accessibility measure, in which the target destinations are assumed to be perfect substitutes for each other.
18. Accessibility reports and queries can be used to determine the total number/attractiveness of the relevant facilities or services (e.g. jobs, schools, GPs, hospitals, food shops etc) k , located within the defined journey time/distance/cost/generalised threshold C of the nearest origin zone of type u (equation 5).

$$A(A^k, u, C) = \sum_{j=1}^J A_j^k \times \partial_j^{uC} \quad (5)$$

Where,

$A(A^k, u, C)$

The total resident relevant facilities or services of type k , which are located within the travel threshold C of the nearest origin location of type u . Indicator range $0 - \sum_{j=1}^J A_j^k$ inclusive.

A_j^k

The total number or attractiveness of the opportunities

located in destination zone j .

$\partial_j^{u^C}$

An indicator function which assumes the following values:
 1 - Unity if destination zone j is within the travel threshold C of at least one origin zone location of type u .
 0 - Zero otherwise.

19. An additional formulation of this expression is:

$$a(A^k, u, C) = \frac{A(A^k, u, C)}{\sum_{j=1}^J A_j^k} = \frac{\sum_{j=1}^J A_j^k \times \partial_j^{u^C}}{\sum_{j=1}^J A_j^k} \quad (6)$$

Where,

$a(A^k, u, C)$

The proportion of the relevant facilities or services of type k which are situated within the travel threshold C of the nearest origin location of type u . Indicators range 0 - 1 inclusive.

20. The aforementioned expressions represent an origin based threshold (cumulative opportunity) accessibility measure, in which the target destinations are assumed to be perfect substitutes for each other.
21. It should be noted that equations 3-6 inclusive represent overall destination and origin based threshold (cumulative opportunity) accessibility measures reported for an entire study area and thus cannot be easily mapped within a geographical information system (GIS). The following series of threshold based network accessibility indicators allow for presentation of accessibility indicators within a GIS environment.
22. Accessibility reports and queries can be used to determine the total relevant resident population type s located within the defined journey time/distance/cost/generalised threshold C of the activity location j (equation 7).

$$P(R^s, C)_j = \sum_{i=1}^{i=l} R_i^s \times \partial_{ij}^C \quad (7)$$

Where,

$P(R^s, C)_j$

The total resident population in segment s which lives within the travel threshold C of the destination activity location j . Indicator range 0 - $\sum_{i=1}^l R_i^s$ inclusive.

∂_{ij}^C

An indicator function which assumes the following values:
 1 - Unity if the journey from i to j is within the travel

threshold C .

Please note that this indicator function specifically refers to a specific I-j pair i.e. an origin-destination combination, whilst $\partial_j^{u^C}$ used earlier indicates that the destination j is within the threshold C of any origin location of type u .

0 - Zero otherwise.

23. An additional formulation of this expression is:

$$p(R^s, C)_j = \frac{P(R^s, C)_j}{\sum_{i=1}^I R_i^s} = \frac{\sum_{i=1}^I R_i^s \times \partial_{ij}^C}{\sum_{i=1}^I R_i^s} \quad (8)$$

Where,

$p(R^s, C)_j$ The proportion of the resident population in segment s which lives within the travel threshold C of the destination activity location j . Indicators range 0 - 1 inclusive.

24. The aforementioned expressions (equations 7 and 8) represent a destination based threshold (cumulative opportunity) accessibility measure associated with a destination location j , which can be contoured or spatially or thematically mapped within a GIS for various j .

25. Accessibility reports and queries can be used to determine the total relevant facilities or services (e.g. jobs, schools, GPs, hospitals, food shops etc) k , located within the defined journey time/distance/cost/generalised threshold C of a particular origin zone i (equation 9).

$$A(A^k, C)_i = \sum_{j=1}^J A_j^k \times \partial_{ij}^C \quad (9)$$

Where,

$A(A^k, C)_i$ The total relevant facilities or services of type k , which are situated within the travel threshold C of the origin location i . Indicator range 0 - $\sum_{j=1}^J A_j^k$ inclusive.

26. An additional formulation of this expression is:

$$a(A^k, C)_i = \frac{A(A^k, C)_i}{\sum_{j=1}^J A_j^k} = \frac{\sum_{j=1}^J A_j^k \times \partial_{ij}^C}{\sum_{j=1}^J A_j^k} \quad (10)$$

Where,

$a(A^k, C)_i$ The proportion of the relevant facilities or services of type k which are situated within the travel threshold C of the origin location i . Indicators range 0 - 1 inclusive.

27. The aforementioned expressions (equations 9 and 10) represent an origin based threshold (cumulative opportunity) accessibility measure associated with an origin i , which can be contoured or spatially or thematically mapped within a GIS for various i .
28. Simple accessibility ratio indicators are to be formed by determining the ratio of travel time/distance/cost/generalised cost for one or more modes of transport relative to another transport mode or for one segmented person type relative to another segmented person type etc. This can be undertaken on an origin/destination point basis, output area, super output area, ward, district, county/PTE or a user defined study area as well as on the basis of point relative to a regional average.

CONTINUOUS MEASURES

29. These indicators combine both the attractiveness of the service or opportunity under consideration such as the number of jobs, retail floor space and the disbenefit or disutility associated with travel. These integrated measures are closely related and encompass:
 - Hansen/gravity measure (defined within equations 11 and 12)
 - Relative Hansen/gravity measure (defined within equations 13 and 14)
 - Simple utility or logsum measure (defined within equations 15 and 16)

HANSEN/GRAVITY MEASURES:

30. Accessibility reports and queries can be used to determine the Hansen/gravity measure of the relevant population type s with respect to the activity location j (equation 11).

$$H(R^s, s)_j = \sum_{i=1}^I R_i^s \times \exp(-I^s \cdot c_{ij}^s) \quad (11)$$

Where,

$H(R^s, s)_j$ The weighted number of the resident population in segment s relative to the destination activity location j .

Indicator range $0 - \sum_{i=1}^I R_i^s$ inclusive.

I^s A positive coefficient which defines the influence of travel time/distance/cost/generalised cost on travel for persons of type s .

31. Equation 11 represents a destination based Hansen/gravity accessibility measure associated with a destination location j , which can be contoured or spatially or thematically mapped within a GIS for various j .

32. Accessibility reports and queries can be used to determine the Hansen/gravity measure of the relevant facilities or services (e.g. jobs, schools, GPs, hospitals, food shops etc) k , with respect to the origin zone location i for the population segment s (equation 12).

$$H(A^k, s)_i = \sum_{j=1}^J A_j^k \times \exp(-I^s \cdot c_{ij}^s) \quad (12)$$

Where,

$H(A^k, s)_i$ The weighted number of relevant facilities or services k for population in segment s relative to the origin zone location i .

Indicator range $0 - \sum_{j=1}^J A_j^k$ inclusive.

A_j^k The attractiveness or total number of facilities or services of type k located at destination zone location j .

33. It is possible to formulate a weighted origin based Hansen/gravity accessibility measure as follows:

$$W(A^k, R_i^s, s)_i = R_i^s \times H(A^k, s)_i = R_i^s \times \sum_{j=1}^J A_j^k \times \exp(-I^s \cdot c_{ij}^s) \quad (13)$$

Where,

$W(A^k, R_i^s, s)_i$ The weighted number of relevant facilities or services k for population in segment s relative to the origin zone location i , weighted by total resident population in segment s

Indicator range $0 - R_i^s$ inclusive.

34. Equations 12 and 13 respectively represent an origin based Hansen/gravity and weighted Hansen/gravity accessibility measures associated with an origin location i , which can be contoured or spatially or thematically mapped within a GIS for various i .

RELATIVE HANSEN/GRAVITY MEASURES:

35. Accessibility reports and queries can be used to determine the Relative Hansen/gravity measure of the relevant population type s with respect to the activity location j (equation 14).

$$h(R^s, s)_j = \frac{\sum_{i=1}^I R_i^s \times \exp(-I^s \cdot c_{ij}^s)}{\sum_{i=1}^I R_i^s} \quad (14)$$

Where,

$h(R^s, s)_j$ The weighted proportion of the resident population in segment s relative to the destination activity location j .
Indicator range 0 - 1 inclusive.

36. Equation 14 represents a destination based Relative Hansen/gravity accessibility measure associated with a destination location j , which can be contoured or spatially or thematically mapped within a GIS for various j .
37. Accessibility reports and queries can be used to determine the Relative Hansen/gravity measure of the relevant facilities or services (e.g. jobs, schools, GPs, hospitals, food shops etc) k , with respect to the origin zone location i for the population segment s (equation 15).

$$h(A^k, s)_i = \frac{\sum_{j=1}^J A_j^k \times \exp(-I^s \cdot c_{ij}^s)}{\sum_{j=1}^J A_j^k} \quad (15)$$

Where,

$h(A^k, s)_i$ The weighted proportion of relevant facilities or services k for population in segment s relative to the origin zone location i .
Indicator range 0 - 1 inclusive.

38. Equation 15 represents an origin based Relative Hansen/gravity accessibility measure associated with an origin location i , which can be contoured or spatially or thematically mapped within a GIS for various i .

SIMPLE UTILITY / LOGSUM MEASURES:

39. Accessibility reports and queries can also be undertaken using the logsum measure of accessibility. This provides an indication of the "expected maximum utility" for the individual/population segment s resident in origin zone location i of the available choice set encompassing a set of opportunities A available in the Base case O with differing levels of (dis)utility. A simple logsum formulation is presented below (equation 16).

$$A(LS)_i^{sO} = \frac{1}{-I_c^s} \left(\ln \left\{ \frac{\sum_{j=1}^J A_j^O \times \exp(-I_c^s \cdot c_{ij}^{sO})}{\sum_{j=1}^J A_j^O} \right\} \right) \quad (16)$$

$A(LS)_i^{sO}$	The logsum measure of accessibility to opportunities A in the Base case O for the resident(s) of segment s living in zone i .
A_j^O	The attractiveness or weighting of the opportunity located at destination zone j in the Base case O .
c_{ij}^{sO}	The generalised cost of travel from i to j for persons of segment s in the Base case O .
I_c^s	A coefficient defining the influence of generalised cost on travel for persons of segment s .

40. Equation 16 represents an origin based logsum measure associated with an origin location i , which can be contoured or spatially or thematically mapped within a GIS for various i . As structured the expression cannot be aggregated for a series of origin locations to arrive at a value for the study area or region as a whole.

SIMPLE TIME CONSTRAINED MEASURES:

41. In the simple time constrained analysis the user specifies the desired arrival time T_p^{ks} and the maximum desired wait time threshold at the destination zone location w of the individual type s at the destination facility or service of type k . The software tool routing algorithm then proceeds to determine the departure time of the individual T_d^{js} given the selected mode(s) of travel such that the minimum journey time is selected subject to the individual arriving before the preferred arrival time T_p^{ks} . If the individual arrives at a facility location before it opens T_o^j then the individual must wait for the facility to open. This wait time w_j^s is defined by $w_j^s = T_o^j - T_d^{js}$. Accessibility reports and queries can be used to determine the Simple Time Constrained Hansen/gravity measure of the weighted

number/attractiveness of the relevant facilities or services (e.g. jobs, schools, GPs, hospitals, food shops etc) k , at the time of arrival with respect to the origin zone location i for the population segment s (equation 17).

$$H(A_j^k, s, T_p^{ks})_i = \sum_{j=1}^J A_j^k \times \partial_j^s \times \exp(-I_t^s \cdot (t_{ij}^s)) \quad (17)$$

Where,

$H(A_j^k, s, T_p^{ks})_i$ The weighted number/attractiveness of relevant facilities/services which are open at the time of arrival for the resident population segment s situated at the origin location i .

Indicator range $0 - \sum_{j=1}^J A_j^k$ inclusive.

T_p^{ks} The preferred arrival time of the individual type s at the destination type k .

T_a^{js} The actual arrival time of the individual type s at the destination zone location j .

T_o^j The opening time of the destination zone location j .

T_c^j The closing time of the destination zone location j .

∂_j^s An indicator function which assumes the following values:
1 - Unity if the actual arrival time T_a^{js} satisfies the following condition $T_o^j \leq T_a^{js} \leq T_c^j$.

0 - Zero otherwise.

I_t^s A coefficient (positive in magnitude) which defines the influence of travel time on travel for persons of type s .

42. The aforementioned expression (equation 17) represents an origin based Simple Time Constrained Hansen/gravity accessibility measure associated with an origin location i , which can be contoured or spatially or thematically mapped within a GIS for various i .

43. Accessibility reports and queries can be used to determine a Simple Time Constrained Relative Hansen/gravity measure of the proportion of weighted facilities or services (e.g. jobs, schools, GPs, hospitals, food shops etc) k , available at the time of arrival with respect to the origin zone location i for the population segment s (equation 18).

$$h(A_j^k, s, T_p^{ks})_i = \frac{H(A_j^k, s, T_p^{ks})_i}{\sum_{j=1}^J A_j^k} = \frac{\sum_{j=1}^J A_j^k \times \partial_j^s \times \exp(-I_t^s \cdot (t_{ij}^s))}{\sum_{j=1}^J A_j^k} \quad (18)$$

Where,

$h(A_j^k, s, T_p^{ks})_i$ The proportion of relevant weighted facilities/services which are open at the time of arrival for the resident population segment s situated at the origin location i .

Indicator range 0 - 1 inclusive.

44. The aforementioned expression (equation 18) represents an origin based Simple Time Constrained Relative Hansen/gravity accessibility measure associated with an origin location i , which can be contoured or spatially or thematically mapped within a GIS for various i .

ACCESSIBILITY RATIOS

45. This type of indicator involves dividing the accessibility index results of an input results file at an individual grid point level by the corresponding results of another input point based results file with the individual grid point ratios output to a separate results file. In performing this calculation there is a need to cater for a denominator of zero in which case the grid point in question is either excluded or allocated a nominal value of unity. However it should be noted that both of these options have their limitations. The use of a nominal value such as 0.001 will overcome some of these. One method of achieving such accessibility ratios is for the user to a) specify the name of the output results file b) select an input results file to form the denominator, c) select an input results file to form the numerator.
46. Calculation of the individual point based accessibility ratios is to be undertaken on a point matching basis where the points aggregated must spatially intersect i.e. must be on top of one another. If not they are excluded from the process. Equation 19 defines the calculation of the indicator.

$$(Output\ Results)_{i/j} = \frac{(Input\ Results)_{i/j}^{num}}{(Input\ Results)_{i/j}^{den}} \quad (19)$$

Where,

$(Output\ Results)_{i/j}$ The name of the output results file which is either origin i or destination j based.

$(Input\ Results)_{i/j}^{num}$ The name of the numerator input results file which is either origin i or destination j based.

$(InputResults)_{i/j}^{den}$ The name of the denominator input results file which is either origin i or destination j based.

FACTORED MEASURES

47. The simplest manner for implementing this type of indicator is for the software tool to enable the user of the application to factor and subsequently combine one or more point based spatially referenced results grids of the same type (i.e. spatially matching for all points) and apply an equal multiplication based weighting to each grid point associated with a particular results file. That is the user is asked to a) specify the name of the output results file b) select an input results file, then he/she is asked to c) specify a multiplication factor to be applied to the index results contained within this input results file. The user is presented with the option of selecting another results file and repeating stages b-c in the process. The user is at the input results file selection stage able to stop the process in which the aggregated output results file will consist of grid points each containing the weighted sum of the input results files defined.
48. Summation of the results operates on a point matching basis where the points aggregated must spatially intersect i.e. must be on top of one another. If not they are excluded from the process. Equation 20 defines the calculation of the indicator.

$$(Output\ Results)_{i/j} = \sum_{z=1}^Z (Input\ Results)_{i/j}^z \times y_z \quad (20)$$

Where,

$(InputResults)_{i/j}^z$ The name of the z -th input results file which is either origin i or destination j based.

y_z The multiplication factor applied to the z -th input results file.

Z Total number of input results files to be combined.

Annex: Public Transport Accessibility Levels (PTALs) Plus Index

Calculating Total Access Time

A1 Total access time is made up of a combination of factors: combining the walk time from the point of interest (POI) to the service access point (SAP) and the time spent waiting at the SAP for the desired service to arrive.

Total Access Time = Walk Time + Average Waiting Time

Average Waiting Time

A2 Waiting time is the average time between when a passenger arrives at a stop or station, and the arrival of the desired service. In PTALs passengers are assumed to arrive at the SAP at random.

A3 For each of the routes associated with the selected modes for the PTALs calculation the scheduled waiting time (SWT) is calculated. This is estimated on the basis of half the headway (i.e. the interval between services) so $SWT = 0.5 * (60/Frequency)$.

A4 Thus a 10-minute service frequency (6 buses per hour) would give an SWT of 5 - on average a passenger would have to wait 5 minutes for a bus/train to appear.

A5 To derive the Average Waiting Time, reliability factors are applied to the SWT according to the mode of transport used. The regularity of buses, underground and rail services is affected by a variety of factors, with bus services the worst affected. To allow for reliability additional wait times assumed are 2 minutes for buses and 0.75 minutes for rail services. These values variable and the user can vary these to assess the impact of measures which would improve service reliability.

Calculating Equivalent Doorstep Frequency

A6 The access time is converted to an Equivalent Doorstep Frequency (EDF) where:

$EDF = 30/Total\ Access\ Time\ (minutes)$

A7 This treats access time as a notional Average Waiting Time as though the route was available at the "doorstep" of the selected POI.

Calculating the Accessibility Index for the POI

A8 Summation of the EDF values gives the accessibility index. There are a number of additional factors that should be considered:

- Routes often travel in parallel for some distance so the range and frequency of destinations are likely to be less than that suggested by the indicator.

- Halving the EDF values for all but the most accessible or dominant route for each transport mode compensates for these factors. Transport modes are divided into three groups.
- Buses
- National Rail
- LUL - All metro, services together with light rail and tram services

A9 Thus for a single transport mode the accessibility index (AI) can be calculated using the following formula:

$$AI_{mode} = EDF_{max} + (0.5 * \text{All other EDFs})$$

A10 Calculating the overall accessibility index is a sum of the individual AIs over all modes.

$$AI_{poi} = \hat{a} (AI_{mode1} + AI_{mode1} + AI_{mode2} + AI_{mode3} \dots AI_{mode n})$$

PTALs Plus Index

The final formula given above calculates the PTAI - the Public Transport Accessibility Index. These indices can now be allocated to a series of bands of Public Transport Accessibility Levels (PTALs).