

LOCAL BUS SERVICES MARKET INVESTIGATION

Provisional decision on remedies—erratum

1. The Provisional decision on remedies (PDR) was published on 6 October 2011. In its response to the PDR the Passenger Transport Executive Group (PTEG) made the following comment regarding the calculation of customer detriment set out in Appendix A of the PDR:

While we welcome the CC's decision to widen its analysis to encompass the impact of the AEC on consumer welfare we have reasons to believe [✂]. Our implementation of what we understand to be the methodology proposed by the CC leads to a much higher level of AEC than the lower bound quoted in table 4 of appendix A.

2. Following the comment from PTEG, we noted an error in the equation we had used for the isoelastic demand function used in Appendix A.¹ When this error is corrected the calculation of detriment using the isoelastic demand function does not produce sensible results.²
3. PTEG suggested that a proportional elasticity demand function was a commonly used demand function in transport studies and may be a suitable alternative for the isoelastic demand function.³ A proportional elasticity demand function has the following form:

$$Q = ke^{bP}$$

Restated in prices this equation becomes:

$$P = \frac{1}{b} \ln \left(\frac{Q}{k} \right)$$

4. Where Q is the quantity of bus travel demanded, k is a constant, e is the mathematical constant,⁴ b is a constant and P represents the level of bus fares. The elasticity of demand at current prices is given by the term bP. An important property of demand functions of this type is that the elasticity of demand changes in proportion to the price.⁵
5. The consumers' surplus under a proportional demand function is given by the following equation:

¹ In Appendix A we used the following form of an isoelastic demand function: $P=kQ^e$. The correct form of this function should have been $P=kQ^{1/e}$.

² The elasticity of demand is equal to -0.4 , consequently the exponent, $1/e$, is equal to -2.5 . Because the value of the exponent in the corrected demand function is greater than 1 in magnitude the integral does not converge. This results in implausibly large values for the consumers' surplus.

³ We note also the discussion in section 2.1 of the *University of Leeds Concessionary Fares Project, Report 1: Economic Principles*. John Nellthorp, August 2010. <http://assets.dft.gov.uk/publications/research-into-the-reimbursement-of-concessionary-fares/report1.pdf>.

⁴ This is sometimes referred to as Euler's number.

⁵ As prices increase, the elasticity of demand increases in proportion. For example if at current prices the elasticity of demand is -0.4 the proportional elasticity function predicts that at twice current fare levels the elasticity of demand will be -0.8 .

$$CS = \int_0^{Q^*} \frac{1}{b} \ln\left(\frac{Q}{k}\right) dQ - P^*Q^*$$

$$CS = \frac{Q^* \left(\ln\left(\frac{Q^*}{k}\right) - 1 \right)}{b} - P^*Q^*$$

6. We agree that it would be appropriate to use a proportional elasticity demand function in our calculations of detriment. Given that it is not possible to observe the precise shape of the demand curve, it is sensible to test more than one specification for the demand function. In addition, with a linear demand function there are large changes in the elasticity of demand as prices increase. With a proportional demand function the elasticity of demand changes at a slower rate as prices increase. The use of a proportional elasticity therefore helps illustrate the sensitivity of our results to this property of linear demand functions.
7. Using the proportional elasticity demand function we obtain larger estimates of the detriment to customers as a result of the AEC. This is shown in the tables below which are corrected versions of Tables 4 and 5 in Appendix A of the PDR.

TABLE 4 **Consumer detriment estimates using conservative definition of rail and tram overlaps**

<i>Demand function</i>	<i>Consumer detriment (£m per year)</i>		
	<i>Lower bound</i>	<i>Central estimate</i>	<i>Upper bound*</i>
Linear demand	79	110	140
Proportional demand	158	219	281

Source: CC analysis.

*The classification of rail and tram overlaps in this table is conservative; these results cannot therefore be interpreted strictly as an upper bound.

TABLE 5 **Consumer detriment estimates using less conservative definition of rail and tram overlaps**

<i>Demand function</i>	<i>Consumer detriment (£m per year)</i>		
	<i>Lower bound*</i>	<i>Central estimate</i>	<i>Upper bound</i>
Linear demand	104	148	191
Proportional demand	208	295	382

Source: CC analysis.

*The classification of rail and tram overlaps in this table is less conservative; these results cannot therefore be interpreted strictly as a lower bound.